A Recursive Dynamic Computable General Equilibrium Model For Agricultural Policy Analysis in Spain (ORANI-DYN).

Part II: Modifications to the Standard ORANI-G model framework

Documento de Trabajo 11/02

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July 2011

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1. ORANI-G: A brief overview of the model

In part I of this report, a detailed explanation is provided on the many steps required in the data construction, data updating and parameter choices which feed into the ORANI-DYN (ORANI "España") 2005 database. ORANI-DYN is based on the 'generic' ORANI (ORANI-G) model computable general equilibrium (CGE) template, originally designed in the late seventies as a government sponsored policy evaluation tool for the Australian economy. As is typical to all CGE models, ORANI-G is a 'comparative static' representation in that it compares two points in time (i.e., there is no mechanism which examines the path from one point to another). Consequently, there is no explicit temporal mechanism in the model, whilst new capital investment (which does not feed into the accumulation of capital stock in the model time frame) employs a choice between three 'simple' allocation mechanisms.

Common to all CGE models, ORANI is based on market clearing equations (i.e., supply equals demand) for each input and output market; and a series of accounting conventions (e.g., income equals expenditure equals output, zero long run profits in production). These market clearing and accounting equations are supplemented by a series of 'behavioural' equations which characterise the demand and supply responsiveness of agents to changing market conditions (i.e., prices). Given the complexity of CGE frameworks, convenient functional forms (i.e., Cobb-Douglas (CD), constant elasticity of transformation (CET), constant elasticity of substitution (CES)) tend to be favoured over fully flexible functions, since the calibration[5] of the functions (i.e., expenditure shares, cost shares) can be achieved more easily employing the underlying benchmark data.[6] Unfortunately, the downside with convenient functions is that they impose tight restrictions on agent behaviour (i.e., price elasticities equal to -1; unitary income elasticities. For this reason, ORANI employs a linear expenditure system (LES) to characterise private household demands. This function also belongs within the family of 'convenient' functions and is therefore relatively straightforward to calibrate, whilst it allows for a more flexible treatment of income and price elasticity responses in final demands.

A further 'trick' employed in CGE model frameworks is the usage of 'nesting'.[7] Since CES/CET/CD/LES functions lack the behavioural flexibility of more complex functional forms, the assumption of weak separability is employed to partition final and intermediate demands into 'nests' (multi-stage budgeting) based on conventional neo-classical behaviour (utility maximisation, cost minimisation). Thus, the decision to purchase a commodity/input 'i' is made independently of the source (i.e., domestic vs. imported) from

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[5] Calibration involves the calculation of the parameter values of a mathematical function to replicate the existing benchmark data flows.

[6] In the case of the CES and CET functions, extraneous estimates of the elasticity of substitution/transformation are also required to complete this process.

[7] For the interested reader, appendix A discusses the issue of nesting examining issues of separability and aggregation properties.

which the commodity is purchased. In each nest, the user is free to employ an 'appropriate' elasticity parameter, although this approach is restricted by the limited availability of elasticity estimates in the literature.

In order to ensure a solution, the number of equations and endogenous variables in the model system must be equal. Thus, a certain number of variables must be held exogenous in order to ensure correct 'closure'. Typically, exogenous variables are limited to productivity variables, factor endowments or tax/subsidy variables. In addition, the choice of closure also forms some maintained hypothesis about the macroeconomic assumptions relating to the economy. These issues are discussed further in section 9 of this report.

As a final comment, the representation of the ORANI-G model and the modified version presented in this document is predominantly in linear terms. Horridge (2003) provides a useful explanation on the percentage change approach (linearised) approach to CGE modelling, whilst appendix B provides a detailed discussion on the derivation of linearised demand functions employing a stylised nesting structure. The following is only intended as an overview of the standard ORANI-G model. For a full and accessible discussion, the reader is encouraged to consult Horridge (2003).

1.1 The Theoretical Structure of ORANI-G

1.1.1 Production

Industries are assumed to be perfectly competitive profit maximizers. This means that input demand is consistent with cost-minimising (Hicksian) behaviour, as well as a nested revenue-maximising output problem to allocate production by commodities and by local/export destinations.

Examining (Hicksian) input demands, each industry minimises costs by choosing the input mix, subject to a three-tier constant-returns-to-scale input technology (see Figure 1). At the top level, it is assumed that intermediate commodity composites, primary-factor composites and 'other costs' are combined using a Leontief function. In the second level of the nest, Hicksian demands for domestic and imported intermediate inputs are subject to a CES (constant elasticity of substitution) production function. The demands for land, labour and capital are also derived by minimising the cost of primary factor composites formed according to CES technology. The bottom level of the input technology is only applicable to labour. As in the case of the second level, composite labour is a CES aggregate of 'skilled' and 'unskilled' labour. The demands for labour in the two skill categories are derived by minimising labour costs subject to this technology. On the output side, a two-nested revenue maximisation problem assigns production by different commodities and subsequently between domestic and export goods. The choice on the composition of output is made subject to a CET (constant elasticity of transformation) production frontier. This multi-production decision is shown at the top of the production tree in Figure 1.

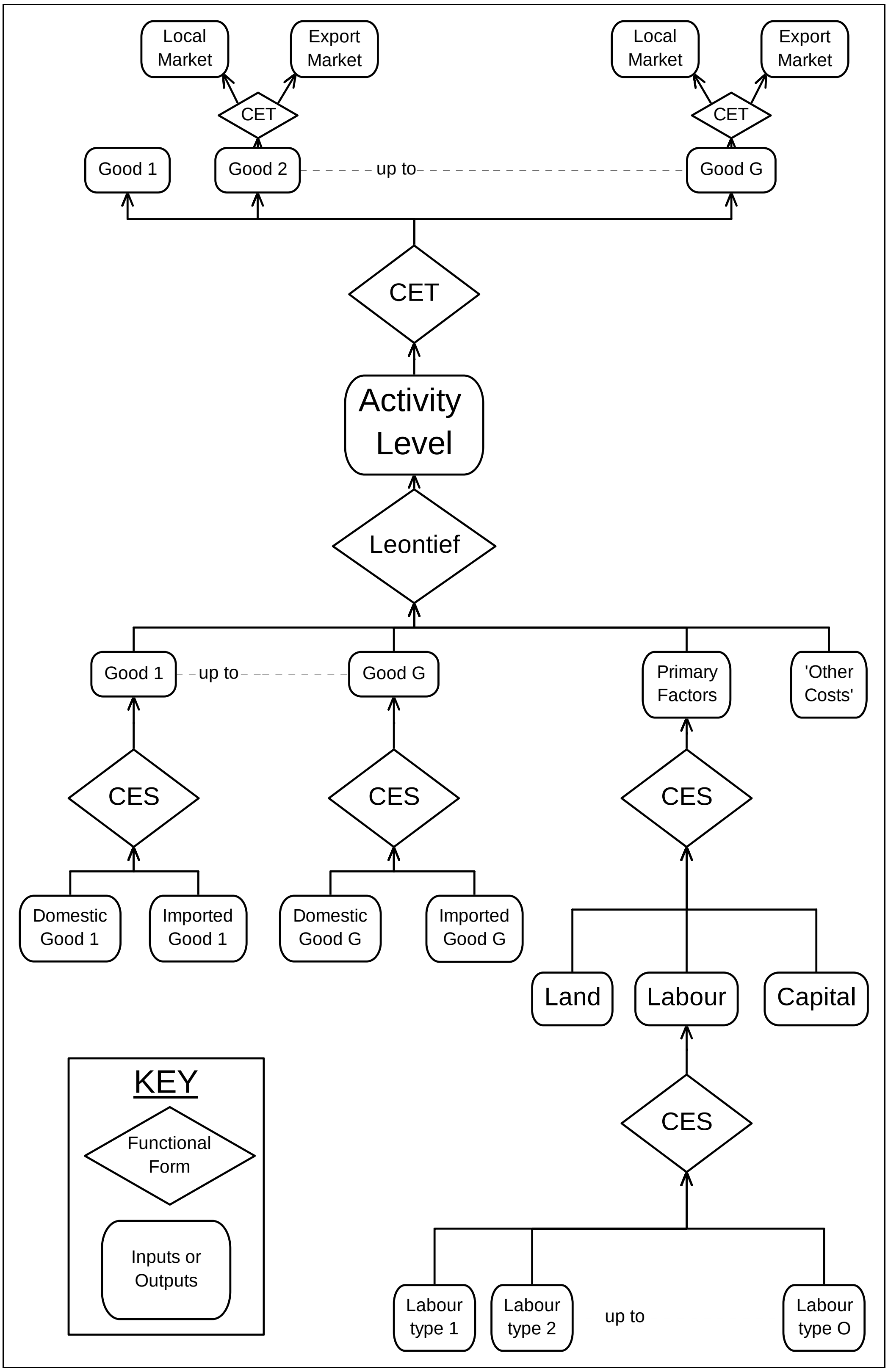

Figure 1: The production structure of ORANI-G:
Source: Horridge (2003)

1.1.2 Private Final Demand

In ORANI-G, final demands stem from four main sources: household consumption, investment/capital creation, government consumption and exports. This is also the classification of final demand adopted in input-output tables, the main source of the model database.

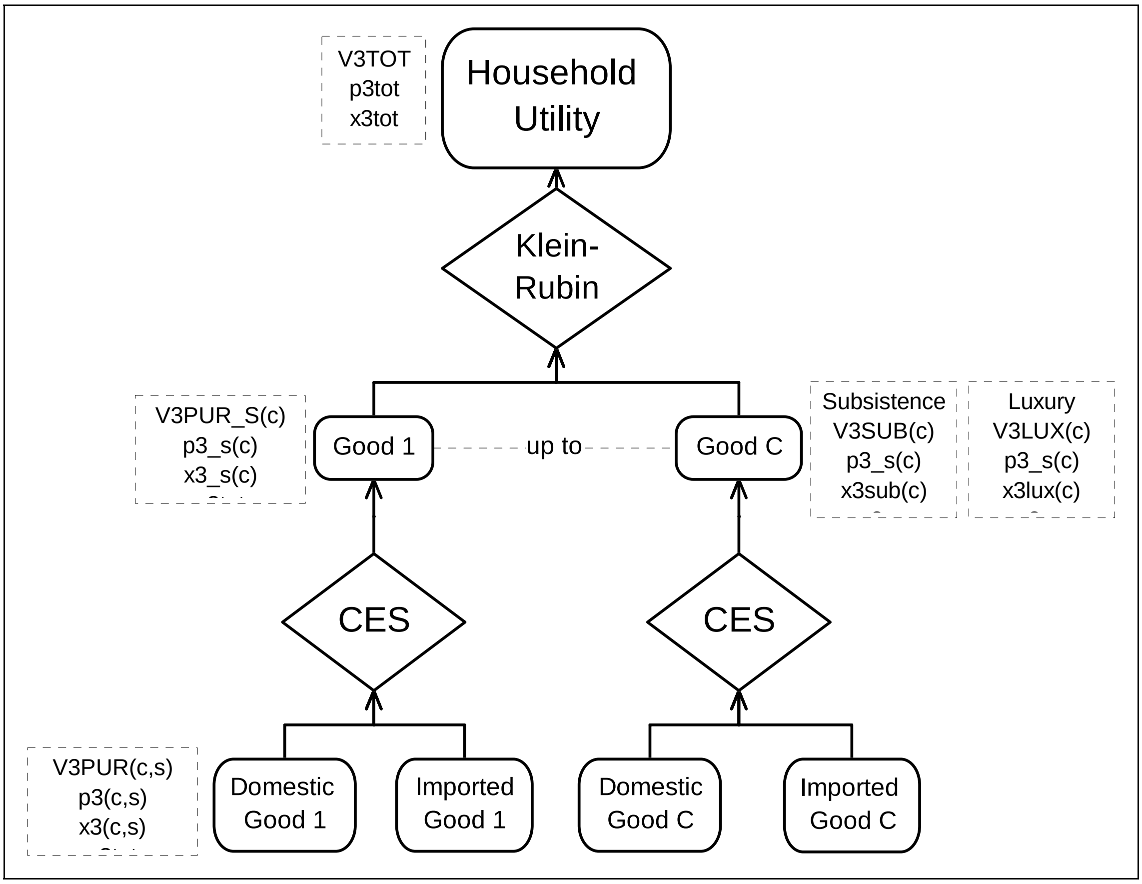

Figure 2. Private Demands in ORANI-G
Source: Horridge (2003)

Private final demands are characterised by a single representative household. The purchase of final demands by source (i.e., domestic or imported) is conditional on a CES function (as in the production nest), whilst 'composite' good final demands are determined via maximisation of a Linear Expenditure System (LES)[8] utility function subject to a budget constraint (see Figure 2). As noted above, the LES function is preferable to CES or CD functions since it permits calibration to non unitary income and price elasticities of demand. This is of particular relevance when characterising (income inelastic) food demands.

1.1.3 Investment Final Demands

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[8] Also known as a 'Klein Rubin' or 'Stone Geary'. This function is clearly discussed in Horridge (2003)

Figure 3 illustrates the nesting structure for capital creation. A new unit of fixed capital used in industry 'j' is constructed according to a two-tiered technology. At the top level, industry minimises cost by choosing the composite goods subject to a Leontief production function, implying that all composite goods are used in fixed proportions. At the next level, substitution between domestic and imported goods is possible (Armington, 1969). It is assumed that primary factors are not employed in capital goods creation.

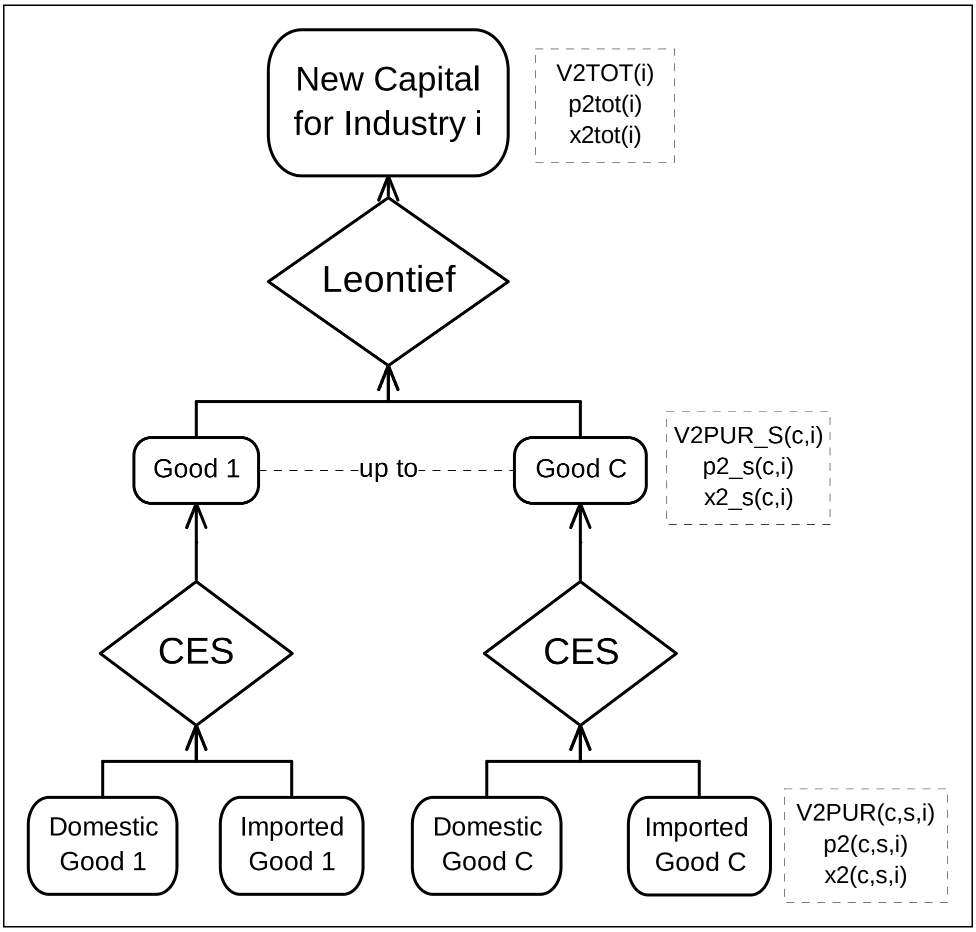

Figure 3: Investment demands in the ORANI-G model.
Source: Horridge (2003)

In the absence of a detailed 'dynamic' investment mechanism, ORANI-G offers the modeller a choice of three potential investment allocation mechanisms. In rule 1 (model variable 'finv1(i)' exogenous), the accumulation of capital goods in industry 'i' to capital stock in industry 'i' is a direct function of changes in the rate of return in industry 'i' (defined as the ratio between changes in the rental price of capital and the unit price (average cost) of a unit of capital good construction) relative to the economy-wide rate of return (variable invslack).[9] In rule 2 (model variable 'finv2(i)' exogenous), the production of

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[9] In the model, there is no explicit recognition of depreciation, although it is implicit in the sense that the rent on capital services is a gross rent figure (i.e., prior to depreciation).

capital goods in industry ‘i’ is directly proportional to the economy wide increase in capital goods production. This rule is more appropriate in those industries where investment is determined by government policy. In rule 3 (model variable ‘finv3(i)’ exogenous), then new capital goods production in industry ‘i’ perfectly shadows the change in capital stock usage in industry ‘i’. It should be noted that in none of these rules is the change in new capital goods fed into the endowment stock of capital (i.e., no capital accumulation). If the user requires an increase in the capital stock, a long run closure should be employed where capital stock is exogenous and shocked.

1.1.4 Export and Public Demands

Export demand by commodity ‘c’ and destination ‘s’ ($X_{c,s}$), is specified by a downward-sloping schedule. Export volume for each commodity is a declining function its price (see also section 5.6).

In the case of public demands, through a closure swap, the user may either specify an exogenous increase in public spending, or assume that public expenditure moves in tandem with changes in private household expenditure.

1.1.5. Demand for ‘Indirect’ Margin Services

Indirect margin services of domestic origin (i.e., wholesale, retail, transport etc.) are used to facilitate the flow of domestic and imported commodities to agents. These demands are assumed to be in direct proportion to the commodity flows with which each specific margin is associated. Note, that the model has nothing to say about international margin services which facilitate the flow of imported commodities from their countries of origin to the point of entry within the domestic economy.

1.1.6. The Price System

ORANI-G distinguishes two types of prices: basic values and purchaser’s prices. For domestically produced goods, basic value is defined as the producer price, excluding commodity taxes and margins used to deliver goods to the final consumer. For imported goods, basic value is the the cost insurance freight (c.i.f.) price received by importers including the tariff, but excluding commodity taxes and margins used to deliver goods to end users. That is, the ‘landed duty-paid’ price. Purchasers’ prices for both imported and domestically produced commodities are the basic prices plus sales taxes and margin costs. In the case of exports, the purchaser’s prices include the margins and subsidy costs, thereby representing ‘free on board’ prices (fob).

In deriving equations representing the model’s pricing system, the following simplifying assumptions are adopted. Long run zero profits occur in all productive activities. Moreover, the basic price is assumed uniform for all users and producing industries. This assumption implies that if a difference in purchasing prices exists across users, this is entirely due to the differences in the sales tax and margin costs. Since constant returns to scale are assumed, the per unit cost and per unit revenue are independent of output level, being influenced only by the level of technology and the prices of commodities. With the above assumptions, the basic prices per unit of industry output equals the total payment for the inputs needed to produce one unit of output.

1.1.7. Market Clearing Equations

For domestically produced commodities, the total supply is driven by the sum of demands for (i) intermediate inputs to current production; (ii) capital creation; (iii) households' consumption; (iv) exports (v) government purchases; and for (vi) margin services.

Over the last 15-20 years, with significant developments in computational power the ORANI-G model has evolved in terms of its complexity whilst retaining a high degree of flexibility. Indeed, the standard data resembles quite closely input-output (IO) national accounts, which makes the model accessible to those researchers interested in building their own CGE characterisations. Moreover, the model structure can be relatively easily modified (with sufficient knowledge of the underlying microeconomics and programming language) to incorporate additional modelling features.

In the next sections, this report divides the main extensions to the model into key areas:

i. The modelling of energy demands in the production and final demand nests for examining the growing importance of biofuels usage.

ii. The increase in the final demand user accounts to include tourism and NGO demands, as well as the disaggregation of private households to allow the modeller to examine the distributive impacts of agricultural policy

iii. Characterisation of labour and capital usage in agricultural/non-agricultural sectors.

iv. Explicit modelling of primary agriculture to characterise the vagaries of agricultural output, factor and input markets.

v. The recursive dynamic treatment of investment

vi. Real wage adjustments to employment.

vii. The fiscal extension of national government and the proportional link between disposable household income and household consumption

viii. The closure in the dynamic ORANI-DYN model

2. Production nests in ORANI-DYN

The major modification which has been made to the nesting structure for each industry is the modelling of energy demands (see Figure 4). The structure of the production nesting to incorporate energy demands follows that employed in the GTAP-E variant (Burniaux and Truong, 2002) of the well known GTAP global trade model (Hertel, 1997). The authors provide an insightful overview of differing modelling approaches to energy substitution. Of particular focus is the close relationship that exists in the literature between capital and energy demand. For this reason, energy goods are removed as intermediate inputs and transferred into the value added nest. Of particular debate is the issue of whether capital (i.e., energy using equipment) and energy demands should be represented as substitutes or complements. In the short run, constraints on energy using capital arise from technological factors (i.e., 'lumpy investment') and adjustment costs. Thus, changes in energy prices have very little impact on energy using capital goods. However, such 'rigidy' factors disappear in the long run such that greater flexibility occurs in capital-energy substitutability.

Further discussion centers on the separation of electrical and non-electrical forms of energy. It is noted that grouping all energy forms together risks masking the important trend of 'electrification' in an energy economy as observed in the US between 1960-82 (Burniaux and Truong, 2002, pp7). Moreover, and more pertinently from the point of view of this study, primary energies (unlike electricity) can also be used as 'feedstocks' into fertilizer usage (rather than consumed as an energy source). In a similar fashion, crude oil is a feedstock into refined petrol, whilst coke may be used in steel production.

With these developments in mind, the modified representation of the production nest in the model is presented in Figure 4. Thus, in the top nest of the input demands structure, a Leontief function is assumed when assigning aggregate expenditures on composite primary factors and energy (value added), 'other costs' and composite non-energy commodities. For each composite non-energy commodity, an upper and lower Armington nest is employed to subdivide input expenditures into domestic and composite imports, and subsequently imports by origin (EU and non-EU source).[10] The 'value added and energy nest' for each industry is a CES aggregate of labour costs, land costs and a capital-energy composite input. Labour is further subdivided into occupation types employing a CES substitution elasticity. The capital-energy aggregate input is subdivided into capital costs per industry and an energy composite input, subject to a CES technology.

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[10] In the standard ORANI-G model, imports are not disaggregated by source.

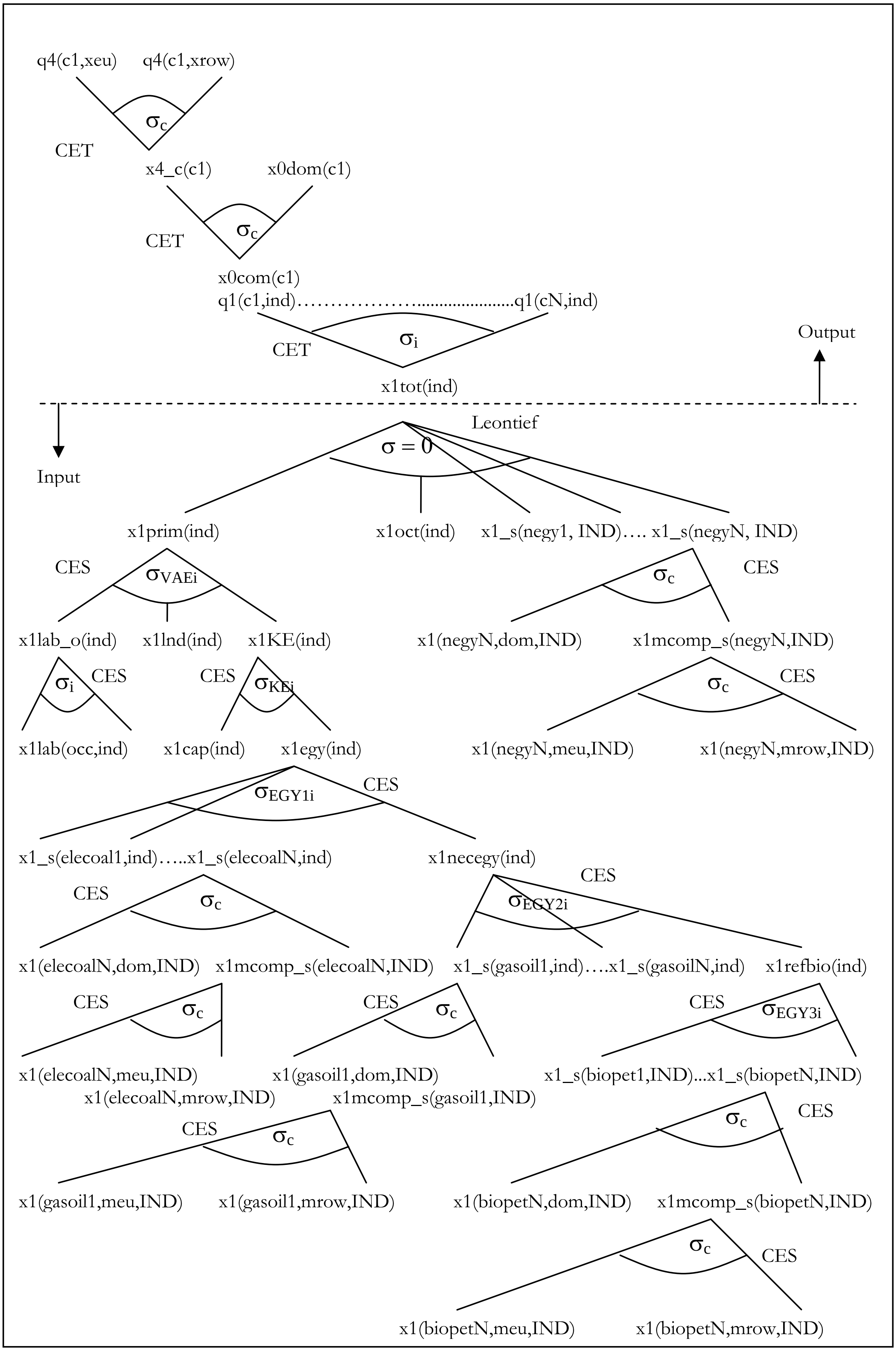

Figure 4: The Production nest in ORANI-DYN.

Note, that the values of the capital-energy nest substitution parameter (σ_{KE}), the composite energy-value added substitution parameter (σ_{EVA}), the cost share of the capital-energy composite in the value added nest (S_{KE}) and the cost share of value added and energy in total output (S_{VAE}) determine whether the overall relationship between capital and energy is substitutable or complementary (Keller, 1980). More specifically, applying the formula derived from Keller (1980, pp83):

$$\sigma_{KE-total} = \left[\frac{\sigma_{KE} - \sigma_{VAE}}{S_{KE}}\right] + \frac{\sigma_{VAE}}{S_{VAE}} \tag{1}$$

Assuming that there are positive substitution elasticities (substitutability) in the upper and lower value added nests, in cases where σ_{KE} is less than σ_{EVA} and S_{KE} is sufficiently small relative to S_{VAE}, then the overall elasticity of substitution between capital and energy ($\sigma_{KE-total}$) may still be negative (complementarity). In this model, σ_{KE} assumes a smaller value than σ_{EVA} (both taken from the GTAP-E document).

The disaggregation of the energy commodity is divided between 'electricity', 'coal' and a 'composite of other energy' types. This composite is split between 'crude oil', 'crude gas' and 'gas' commodities and a petroleum/nuclear and biofuels composite good. The petroleum/nuclear and biofuels composite good consists of petroleum/nuclear commodity, bioethanol from cereals, bioethanol from sugar cane (practically zero) and biodiesel from oilseeds and vegetable oils.

In the case of the each of the CES input demands for specific energy commodities, upper and lower Armington nests (σ_C) are employed to subdivide input costs into domestic and composite import substitutes, where the import composite is further subdivided into imports by region of origin (EU and non-EU imports).

Note that the elasticity of substitution between different energy types in production is also very small (see σ_{EGY1}(i); σ_{EGY2}(i)) and taken directly from the GTAP-E database (calibrated to low price elasticities of demand for energy). In the case of σ_{EGY3}(i), the value is zero, since biofuels are to be blended with petrol usage (i.e., complements) (Birur et al. 2008).

In the top part of Figure 4, the disaggregation of industry activity into multi-product output is controlled employing revenue maximisation criteria subject to a constant elasticity of transformation function (CET) (with an elasticity of transformation of 0.5). Thus, in percentage change terms, employing the equation:

$$q1(c,i) = x1tot(i) + CET(i)\left[p0com(c) - p1tot(i)\right] \tag{2}$$

then an increase in the commodity price compared with the average industry price induces an increase in the production of that commodity 'c' in industry 'i'

The output of commodity 'c' by industry 'i' is translated into commodity outputs via the market clearing equation:

$$x0com(c) = \left[\frac{MAKE(c,i)}{MAKE\_I(i)} \right] \times q1(c,i) \tag{3}$$

where the output of commodity 'c' is the sum of the industry 'i' output shares multiplied by their respective outputs of commodity 'c'. Employing a revenue maximising CET structure, the output of commodity 'c' is divided between exports, 'x4_c(c)' and domestic demands, 'x0com(c)'. Subsequently, a further CET function assigns exports between EU and non-EU export trade routes.

3. Final demands in ORANI-DYN - Private households, Tourism and NGOs

In comparison with the standard ORANI model, ORANI-DYN includes two additional final demand accounts – inbound tourism subdivided by foreign and domestic tourists; and non-governmental organisation (NGO) demands for commodities. In addition, changes have been made to the private household nests to accommodate two modelling extensions: The first is the incorporation of multiple households, stratified by income groups. The second extension relates to the treatment of energy demands, in particular, the potential for substitutability between petroleum and biofuels 'at the pump'.

Employing concepts of weak homothetic separability, additional layers of nesting facilitate greater detail in modelling substitution possibilities in the final demand structure (compare Figure 5 with Figure 2). More specifically, in Figure 5, ORANI-DYN incorporates a split between composite energy and non-energy demands in the linear expenditure system nest. Subsequently, CES energy demands are disaggregated between non-biofuels and petroleum products, and biofuels and petroleum products. In each case these are further subdivided employing upper and lower CES Armington nests. Thus, the upper nest contains a domestic commodity and the composite import substitute; which in the lower nest is subdivided between EU and non-EU import routes. Indeed, the subdivision of EU and non-EU import trade constitutes an additional nest in the ORANI-DYN model compared with the standard ORANI. A discussion of the elasticity estimates in each of the nests is provided in section 19.1 in part I of this report.

Whilst the CES elasticities are assumed constant over all 'h' private households (there are eight in total), the LES expenditure elasticities in the top nest are differentiated by disposable income grouping.[11] Once aggregate household demands, 'x3tot(h)', are derived, these are aggregated over all 'h' to determine Spanish private household demand, 'x3tot_h'.

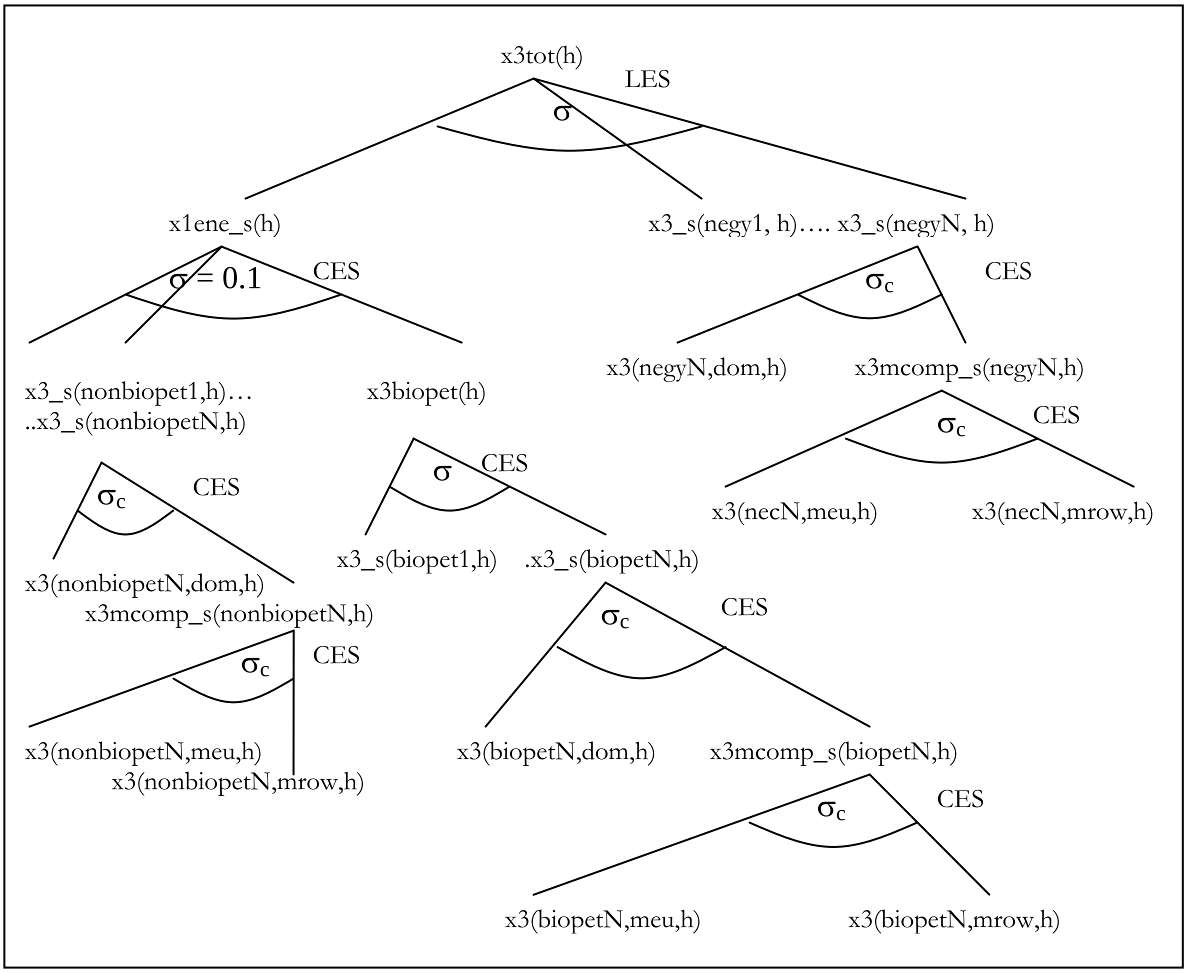

Figure 5: The Private consumption nest for each household 'h' in ORANI-DYN.

'Inbound tourism', is defined as tourism expenditures within the territory of interest. In Figure 3 of part I of the report, the tourism account represents a new column addition in the ORANI-DYN database, labelled as account 7. The structure of 'inbound' tourism demands in the ORANI-DYN model is detailed in Figure 6. Since all of tourism expenditure is effectively a luxury or supernumerary expenditure, the usage of an LES function is not deemed appropriate.[12] In the ORANI-DYN model, the overall composition of commodity demands in inbound tourism is assumed to stay constant over time and changes in line with the fortunes of the tourism industry in Spain. Thus, in the top nest, a Leontief function is employed. Similarly, in the second nest, a Leontief function is also employed between domestic and foreign tourist expenditures on each composite

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[11] A discussion of the relevant expenditure elasticities and Frisch parameters is provided in section 19.1 in part I of this report

[12] The LES function incorporates a subsistence and a supernumerary element to expenditure.

commodity. Domestic and foreign tourists are not seen as substitutes (i.e., assuming that there is 'tourism slack', greater domestic tourist demand does not imply reduced foreign tourist demand, or vice versa). Indeed, employing Leontief, we are assuming that the composition of foreign/tourist demand expenditure remains constant and also moves in direct proportion with the fortunes of the Spanish tourist industry. In the lower nests, Armington CES nests are employed to characterise the substitutability between a domestic commodity and an import composite commodity; and imports from the EU and non-EU regions, respectively. These lower level Armington elasticities are the same as those employed in the private household Armington nests.

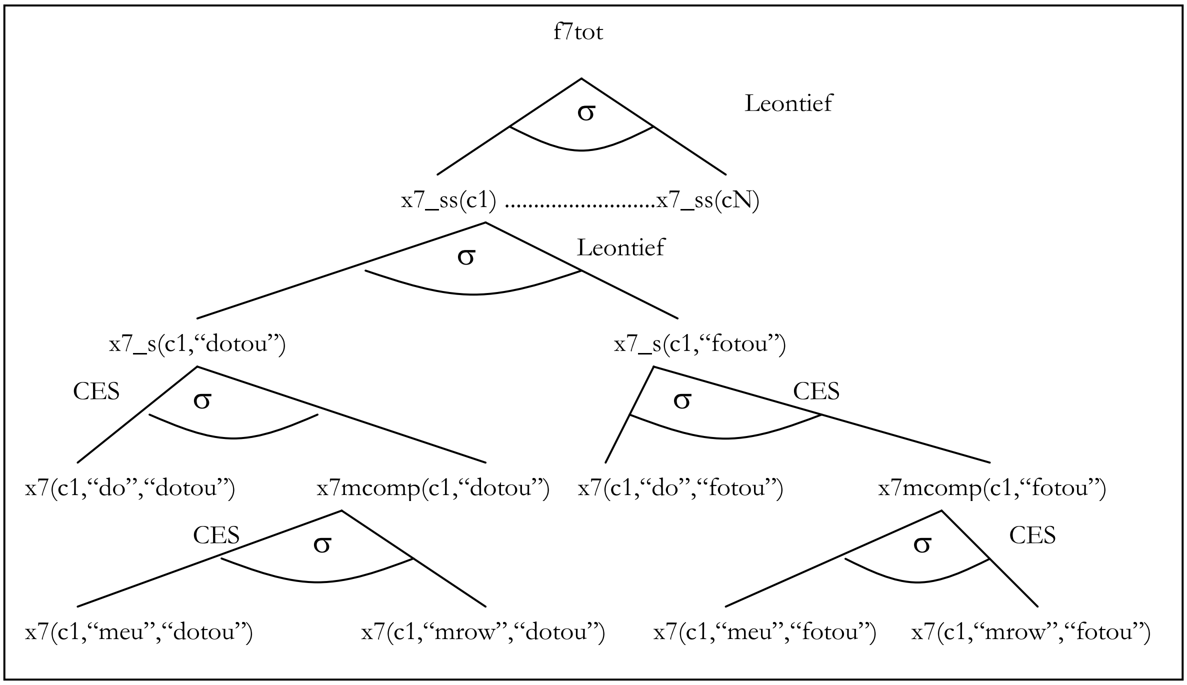

Figure 6: The Tourism nest in ORANI-DYN.

In the ORANI-DYN CGE database represented in Figure 3 of part I of the report, there is also a new account '8' which represents NGO expenditures on commodities. The model treatment for NGO expenditures follows the same simple structure that is used for government (public) demands in the ORANI model. Thus, changes in NGO demands, 'x8(c,s)', can be proportional to a simple exogenous aggregate shock variable (specified by the modeller), or alternatively employing a closure swap, it is possible to model changes in NGO demands in direct proportion to endogenous changes in real aggregate Spanish household demand.

4. Labour and capital transfer in ORANI-DYN

In the standard ORANI-G model, capital and labour may be allowed to move perfectly between using industries 'i'. This implies that the return of capital and labour is equal for each using industry 'i'. In the ORANI-DYN model, labour and capital transfer between the primary agricultural and non-primary agricultural sectors is made 'sluggish' via the usage of a CET function (see Figure 7). The policy implication is that in the real world, there are observed differences in the return to capital (rent) and labour (wages) between the two sub sectors. This concept follows the work on the agricultural variant of the GTAP model ('GTAP-AGR') by Keeney and Hertel (2006). In both cases, the elasticity of transformation in each nest is the same as that employed in the GTAP-AGR model.

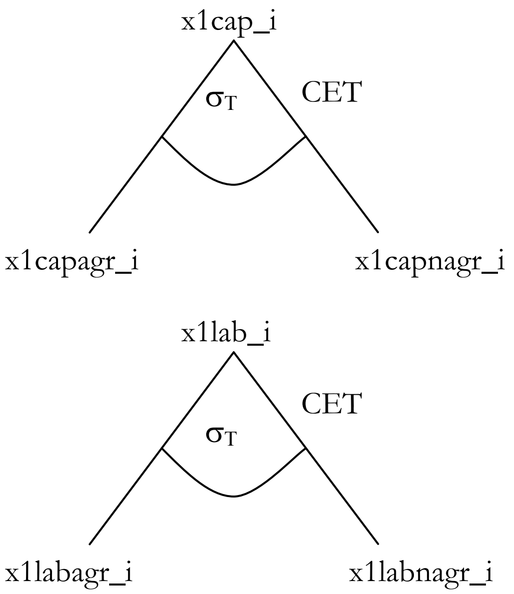

Figure 7: The CET Labour/Capital Allocation between agricultural and non agricultural sub sectors.

5. Explicit Modelling of Primary Agriculture

5.1. Production Quotas[13]

In the ORANI-DYN database, milk and sugar production are divided into 'upstream' production (raw milk; sugar beet/cane) and 'downstream' production (dairy, sugar processing). The implementation of quotas is imposed on the upstream part of the supply chain.

In the standard GTAP model treatment (van Meijl and van Tongeren, 2002), a quantitative restriction is characterised as a simple closure swap. Thus, industry output, is

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[13] In ORANI-DYN, sugar and milk quotas use the same microeconomic framework. In the context of sugar, the advantage of this approach is that it does correctly characterise quota as an additional factor of production (read section 5.1 for further discussion) and also captures the binding/non-binding status of the quota mechanism. However, this treatment does not capture all of the nuances of the EU sugar policy, namely, the self financing principle and the A, B and C quota rates/price differentials prior to the 2006 EU sugar reforms.

exogenised and the output tax variable is endogenised. Thus, endogenous changes in the production tax (now inclusive of quota rents) capture the necessary price changes in order to maintain production fixed. There are, however, three issues with this treatment.

1. It does not separate out taxes from quota rents (as they are assumed 'mixed together' in the tax wedge)
2. Characterising the quota rents as a tax does not capture the fact that the quota is an essential additional factor of production (that is, without quota, it is impossible to produce) and therefore, the rents accrued are analogous to a factor payment.
3. Given that output (qo) is exogenous, we are implicitly assuming that the quota is always binding, when this may not necessarily be the case.

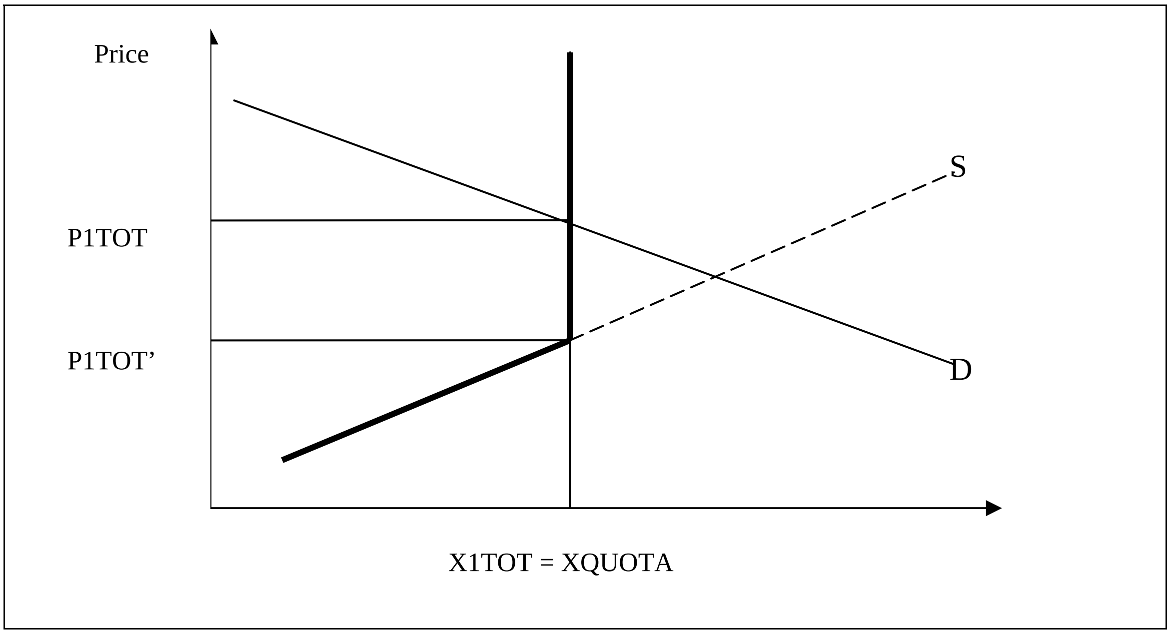

Figure 8: Simple analytics of a quota

For these reasons, ORANI-DYN takes advantage of GEMPACK's complementary[14] code (Bach and Pearson, 1996) which allows an endogenous regime switch between binding and non-binding status in the quota and the modelling techniques of Lips and Rieder (2005). In the ORANI-DYN model database, the rent value is inserted as an additional factor of production (with intermediate and primary costs, production taxes and primary factor subsidies). This implies that given zero profits, changes in rents impact on final prices. Lips and Rieder (2005) employ such a modelling characterisation since they argue that, "producers get the quota rent in the form of a higher producer price and not as a transfer payment" (pp3). Also note that in the ORANI-DYN model, agricultural output is characterised by sector output, not by individual farming units. Thus, the quota must be

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[14] Complementarity equations are effectively inequality constraints

modelled as sector-wide output constraint whilst the purchase/sales of quota between farms cannot be captured in this model, but is implicitly assumed to be efficient.

Examining Figure 8, we see a hypothetical situation where the quota is binding (below the equilibrium) output level. In this case, the quota level of industry output and the current level of industry output are equal such that:

$$XR = X1TOT / XQUOTA = 1 \tag{4}$$

This implies that the shadow price or marginal cost unit price of production is at P1TOT' (governed by the intersection of the supply curve with the quota limit), whilst the market price of production is P1TOT. The difference between these prices is the per unit quota rent. If the supply or demand curve shifted to the left sufficiently, the quota may no longer be binding, such that:

$$XR = X1TOT / XQUOTA < 1 \tag{5}$$

and the quota rent would fall to zero. To characterise this either/or scenario, a complementarity equation is employed into the GEMPACK model code which asserts that if the quota quantity ratio is binding (XR =1), then rent must be zero; whilst a less than binding value (i.e., $0 \leq XR < 1$) implies a positive rent value. Increases/decreases in quota allocations are implemented through increases/decreases to the exogenous variable XQUOTA. Assuming a binding status, a quota increase means that the ratio XR falls, thereby allowing X1TOT to increase endogenously to meet the additional allowable quota, or else the quota is non-binding and rent falls to zero (i.e., the shadow and producer prices are equal). The initial 2005 values of XR and 'rent' are discussed in section 19.2 of Part I of this report.

5.2 Econometric Estimation of the Land Supply Function and implementation into ORANI-DYN

In the standard CGE model treatments, land supply is exogeneous in each region. However, in reality, agricultural land supply can adjust due to the idling of agricultural land or the conversion of land to non agricultural uses. The supply of agricultural land depends on its biophysical suitability, institutional factors (agricultural, urban and nature protection policies) and land price (Tabeau et al., 2006, p.3). Biophysical suitability refers to climate, soil and water conditions that make a plot of land suitable for cultivation. Accordingly, biophysical parameters will define the maximum potentially available land surface that can be used for agricultural purposes (the asymptote in Figure 9). At the outset, the most productive land is used first. With increases in land usage, farmers must employ less productive land implying that the marginal cost of conversion rises, which is reflected in a higher land price. This relationship between land usage and prices gives an upward sloping supply curve (see Figure 9).

Any point along the supply curve is feasible from an agronomic point of view, however, any country/region will be positioned on a specific point, representing the current relative use of land in the agricultural sector. When the region is currently using a low proportion of all the potentially available land, any increase in demand for agricultural land will lead to conversion towards agricultural uses at a modest increase in price (e.g. point A in Figure 9). In this zone of the supply curve, the supply elasticity is relatively higher, and the marginal cost of converting non-agricultural land into agricultural land is relatively lower. However, when a region is currently cultivating most of the available land (e.g. point B in Figure 9), any increase in demand that requires the conversion of the scarce non-used land to agriculture, will lead to the conversion of the least productive land and at a relatively higher marginal cost (land supply elasticity is low).

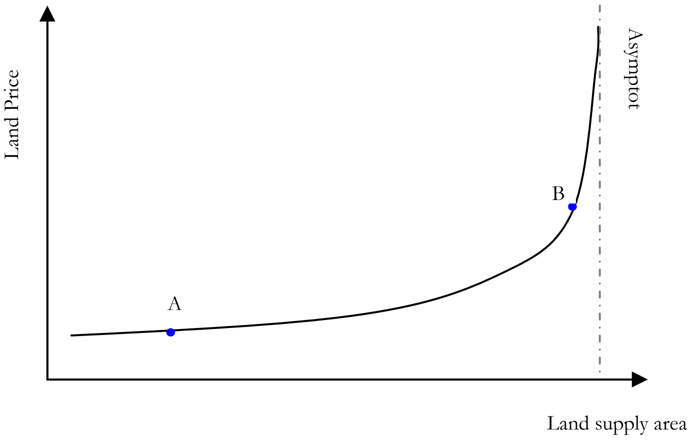

Figure 9: Theoretical agricultural land supply curve

Based on the work of van Meijl *et al.*, (2006) and Tabeau *et al.*, (2006) the land supply function in Figure 9 above takes the functional form:

$$Accumulated\ Area = a - \frac{b}{C_0 + \mathrm{Re}nt^{\rho}} \qquad (6)$$

where ‘a’ is the asymptote or maximum potentially available agricultural land; ‘b’, ‘C’ and ‘ρ’, are estimable parameters, and ‘Rent’ is the price of land.

In the estimation of the land supply function, data are employed on potential agricultural areas and yields provided by the bio-physical model: “Global Agro-ecological Assessment for Agriculture in the 21st Century: Methodology and Results” (GAEZ). IIASA-FAO (RR-02-02). This source combines geo-referenced inventories of data for

Spain on (i) the biophysical characteristics of the land resource, such as soil, terrain slope and climate; with (ii) growing requirements of crops (solar radiation, temperature, humidity, etc.), to calculate the amount of land that may be classified as suitable for producing each crop and the maximum potential and agronomical attainable yield.

The suitability and crop yield estimates are carried out for grid cells of 5 minutes latitude/longitude (equivalent to surfaces of 9.3 km (9.3 km at the equator), and for three alternative generic levels of technology (combination of inputs and management): 'low', using neither fertilizers, pesticides nor improved seeds and using only mechanical tools; 2. 'intermediate', with some use of fertilizers, pesticides, improved seeds and mechanical tools; and 'high', with full use of all required inputs and management practices as in advanced commercial farming. The resulting average attainable yields for each cell, crop and technology are then compared with those obtainable under the same climate and technology but without soil and terrain constraints (the so-called Maximum Constraint-Free Yield).

The potential land in each grid cell for each crop and technology level, is then subdivided into five suitability classes according to attainable yield ranges relative to maximum constraint-free crop yields (GAEZ study, p. 38): 'very suitable' (80-100%), 'suitable' (60-80%), 'moderately suitable' (40-60%) 'marginally suitable' (20-40%), and 'not suitable' (0-20%). Combining the information between suitability classes and technology input levels, results on the extension of land and yield potential for each grid cell (0.5 minutes latitude/longitude) is obtained at the so-called 'mixed input' technology. The procedure of calculus consists of selecting first the extension of land very suitable and suitable at high level of inputs; then the extension of land (of the remaining extent available in the grid-cell) very suitable, suitable or moderately suitable at intermediate level of inputs; and finally, the remaining extensions of land defined as suitable (any degree of suitability) at the low level of inputs (GAEZ study, p.80). Following these three steps but applied only to the crops with the largest extensions, an overall figure of rain-fed land potential is also calculated.

Results of the GAEZ study are published on the web site of IIASA: HYPERLINK: http://www.iiasa.ac.at/Research/LUC/SAEZ/index.html

For each crop, there is a spreadsheet that contains one page for each level of input-technology (low, intermediate, high and mixed). Within each page, land areas and yields are shown for the five land suitability categories, and for each country. For practicability, we have chosen the data sets corresponding to the use of 'mixed levels of inputs'. Likewise, we have chosen data corresponding to the first four suitability categories, from very suitable to marginally suitable.

Crop	Number of crops 'types' in the category
Cereals	(83)
Wheat (spring and winter)	16
Rice (Wetland and dryland)	11
Maize (grain)	13
Silage maize	6
Barley (Hibernating and non-hibernating)	16
Sorghum	7
Millet (pearl and foxtail)	6
Rye (Hibernating and non-hibernating)	8
Roots and Tubers	*(8)*
White potato	4
Sweet potato	3
Cassava	1
Pulses	*17*
	(25)
Oil crops	
Soybean	6
Rapeseed (hibernating and non-hibernating)	8
Groundnut	3
Sunflower	6
Oil palm	1
Olive	1
	7
Fiber crops	
Cotton	7
	(6)
Sugar crops	
Sugarcane	1
Sugar beet	5
Fruit crops	*(1)*
Banana	1
Forage/fodder [1]	*(7)*
Alfalfa	1
Total	*154*

Table 1. List of crops covered by the GAEZ study.

[1] Forage/fodder also includes in the AEZ model Pasture grasses (4 crops) and Pasture legumes (2 crops) for which, however, there are not available data to use in the land supply estimation.

Source: GAEZ (2002, Table 4.1 in p.37) and GAEZ (2002) in www.iiasa.ac.at/research/LUC/SAEZ/in47.htm (10th January 2007), Spreadsheet: c1 to c23.

A degree of precision is needed with respect to the estimates of potential suitable areas for each crop and country. The GAEZ study includes the term 'gross' when referring to potential suitable areas since the land required for non-agricultural uses, such as infrastructure, human settlements or legally protected areas is not subtracted. According to the GAEZ study, in reality, some 10-30% of potentially suitable areas from an agronomic point of view may not be available for agriculture due to other competing uses. The list of the crops for which data on 'gross' potential land and yields is available on the cited web page is presented in Table 1.

To proceed with the estimation of the land supply, first the yields data are sorted in descending order (with the corresponding potentially suitable areas), and second, the ascending area is accumulated. Then, following Tabeau et al., (2006) the variable 'land price' is defined as the inverse of the potential yield (1/yield). Furthermore, for each observation, a relative yield is calculated, dividing each potential yield by the maximum attainable by the region. In this way, the potential relative yield lies between 0 and 1, while the corresponding relative rental rate or land price, will have a minimum of 1. This scaling process helps to infer the relative suitability of each country for each crop, while from an econometrical point of view, scaling contributes to accelerate the convergence to a solution.

Thus, the relationship between the observations on accumulated land area and relative price follows an upward sloping curve (land supply). To improve the fit of the estimated supply parameters (b, C and ρ) to the observed data points,[15] a Maximum Likelihood non linear regression method is employed. A key advantage of Maximum Likelihood is that it can be applied to a wide variety of models (e.g. models with binomial, multinomial, or censored dependent variable). Irrespective of the numerical algorithm used to find a solution to a non-linear model, the maximum likelihood estimator has good asymptotic (for large samples) properties: it is consistent (when the sample size tends to infinity, the expected value of the estimator approaches its true value and its variance tends to zero); asymptotically efficient (the variance of the asymptotic distribution of the ML estimator is smaller than the asymptotic variance of any other consistent estimator); and estimates of the (asymptotic) variances of the estimators as a by-product of the estimation process (Pindyck and Rubinfeld, 1998). For regression models (with a continuous dependent variable), if we make the assumption that the error terms are normally distributed, the maximum likelihood estimators coincide with the various least squares estimators. Therefore, non-linear least squares could also be used. In practice, however, results from both models could differ, but mainly because of the numerical algorithms implemented for each method and by each econometrical package.

The econometric model to estimate then becomes:

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[15] The smaller is the value b, the more inelastic is the land supply curve. The smaller is C, the more elastic is the land supply curve. The smaller is ρ, the more inelastic is the land supply curve.

$$\mathrm{R\_Area}_j = 1 - \frac{b*}{C_0 + \mathrm{R\_Rent}_j{}^p} + \varepsilon_j \qquad (7)$$

where the sub-index j refers to each of the j observations available for each GTAP region: j = 1, …, N, where N is the number of observations and equal to the number of crops times the types of land suitability (with a maximum of 92 observations, 23 crops × 4 land suitability classes); and ε_j is the error term, distributed as a Normal $N(0,\sigma^2)$ (as in non-linear least squares). Then, the likelihood function is specified as:

$$L(\alpha, b^*, C_0, p, \sigma^2) = \prod_{j=1}^{N} \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{1}{2\sigma_2}\cdot\left[\mathrm{R\_Area}_j - \left(\alpha - \frac{b^*}{C_0 + \mathrm{R\_Rent}_j{}^p}\right)\right]^2} \qquad (8)$$

leading to the the log-likelihood:

$$\ell(\alpha, b^*, C_0, p, \sigma^2) = \sum_{j=1}^{N}\left(-\frac{1}{2}\log 2\pi - \log\sigma - \frac{1}{2\sigma^2}\left[\mathrm{R\_Area}_j - \left(\alpha - \frac{b^*}{C_0 + \mathrm{R\_Rent}_j{}^P}\right)\right]^2\right) \qquad (9)$$

In our estimation we use the algorithms that GAUSS uses as default values: the Broyden-Fletcher-Goldfarb-Shanno (BFGS) iteration procedure to obtain the direction δ, and the cubic or quadratic method STEPBT to obtain the step length ρ. As initial values for the parameters (α, b^*, C_0, p) we assign respectively the values (1, 0.05, 0, 0.05). The α-parameter is kept fixed at 1. We apply Weighted Maximum Likelihood estimation where the weight for each observation is the value of $\mathrm{R\_Rent}_j$, as in Tabeau et al. (2006), in order to assign relatively more weight to those observations located at the ends in order to improve the fit of the estimated function to the original data. Finally, among the different alternatives to estimate the covariance matrix of the parameters (standard errors) we use the method implemented by default, which uses the inverse of the Hessian (matrix of the second derivatives of the log-likelihood function).

Writing the flexible non-linear expression for land supply in the GEMPACK model code gives the following levels expression:

$$AREA = 1 - \left[\frac{B}{C + RENT^{\rho}} \right] \tag{10}$$

where AREA is the levels variable for the change in “accumulated area” of land in Spain relative to the asymptote (a ratio which ranges between 0 and 1). The parameters B, C and ρ are read in from a parameter file with estimated values 137.830, 154.367 and 2.272, respectively. The variable ‘RENT’ is the levels real price of land, which is calibrated given knowledge of the aggregated AREA variable and the parameters.

The update for the levels variable RENT is based on the corresponding percentage change linear variable (plandreal) in the model, which is a function of changes in the aggregated price of land in Spain, p1ld_m, and changes in the consumer price index, p3tot_h. In percentage terms:

$$plandreal = pldm\_i - p3tot\_h \tag{11}$$

Given changes in the real land price from the CGE model solution, and knowledge of the parameter values B, C and ρ, the flexible land supply calculates corresponding changes in land supply. Since the model solves in percentage changes, validating the implementation of the estimated land supply function in ORANI is given by checking that calculated land supply elasticities from the CGE model[16] from small incremental shocks are close to the initial single point elasticities calculated from the expression:

$$E^{S} = \frac{\partial Area}{\partial \mathrm{Re}\,nt} \bullet \frac{\mathrm{Re}\,nt_{C}}{Area_{C}} = \frac{\hat{b}^{*} \cdot \hat{p} \cdot \mathrm{Re}\,nt_{C}^{\hat{p}}}{(\hat{C}_{0} + \mathrm{Re}\,nt_{C}^{\hat{p}})\left(\hat{C}_{0} + \mathrm{Re}\,nt_{C}^{\hat{p}} - \hat{b}^{*}\right)} \tag{12}$$

where the circumflex over the parameters indicate the econometrically estimated values.

5.3 Introducing a Multi-Stage CET Function into the Land Market.

In the standard ORANI model, Figure 10 shows how a CET function is introduced to model aggregate land allocation (variable x1lnd_i) across primary agricultural industries ‘i’ (variable q1lnd(i)).[17] The inclusion of a constant elasticity of transformation (CET) function captures the imperfect substitutability between different land types (i.e., land use

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[16] The supply elasticities are simply calculated as the ratio of the percentage change variables x1lnd_i/plandreal

[17] Only the primary agricultural sectors use the land factor in ORANI-DYN.

in different industries implies different land types), although with only one CET nest, the degree of substitutability is equal between different land uses.

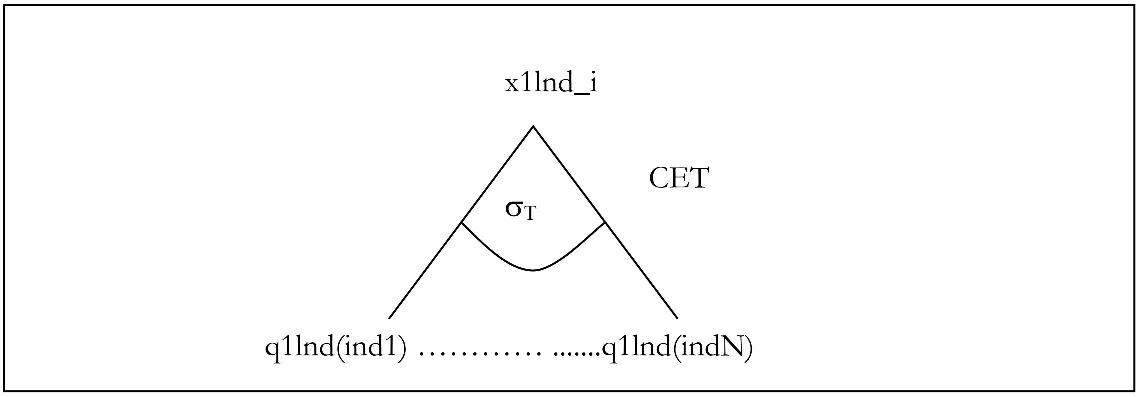

Figure 10: The CET Land Allocation Tree in the Standard ORANI Model.

In ORANI-DYN, we follow the characterisation of Tabeau et al. (2006) who used the allocation structure employed in the OECD's Policy Evaluation Model (OECD, 2003) by assuming that the transferability of land allocation differs by land types. Figure 11 presents the modified allocation structure which is divided into three levels. The top nest controls the supply of land to the composite 'field crops and pastures' (FCP) sector and remaining primary agricultural sectors.

In the second nest, the FCP group is itself a CET aggregate of 'extensive' livestock sectors (cattle, sheep and goats, raw milk), and a composite 'cereal, oilseed, textiles and sugar' crops (COTS). Finally, in the COTS bottom nest, land is allocated between wheat, barley, maize, rice, 'other' cereals, sugar, oilseeds, textile crops and feedcrops.

Using this structure, one may specify an increasing degree of transformation (substitutability) between land types, where the more distinct are the agricultural activities (moving up the tree), the smaller are the elasticities. In the ORANI-DYN model, the CET elasticities in the lowest nest are identical to the standard GTAP model (σ^T=0.5). In the second nest, land substitutability between extensive livestock activities is modelled as more sluggish (σ^T=0.05). Similarly, in the top nest of Figure 11, land substitutability is assumed highly immobile between permanent crops (e.g., fruits sectors, olives, vegetables) and intensive livestock sectors such as pigs and poultry (σ^T=0.005).

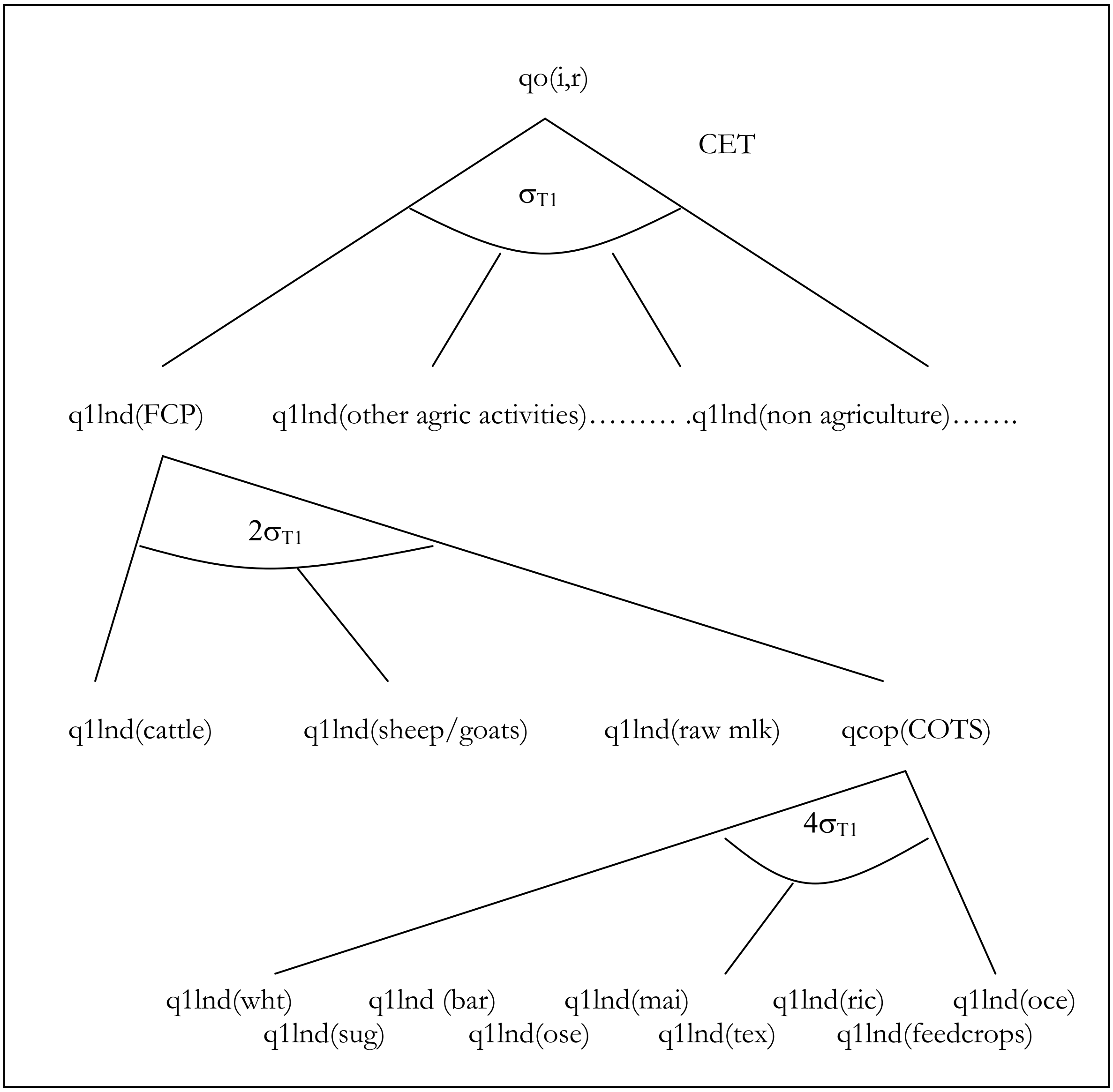

Figure 11: A Modified 3 nested CET Land Allocation Tree.

5.4 The Single Farm Payment (SFP)

The benchmark year for the ORANI-DYN database is 2005, which is one year prior to the implementation of the single farm payment (SFP) in Spain. Thus, in the benchmark year, Spain is still operating under the 'old' Agenda 2000 system of area payment, set aside payments, headage payments, extensification premia etc. Following the characterisation in the GTAP CGE model, these quasi-decoupled payments are characterised as land and capital subsidies[18] in the ORANI-DYN database (see section 17.10 in part I of this report).

To implement the SFP in the post 2005 period, we follow Frandsen et al. (2003). In their paper, the authors characterise the SFP as a uniform land payment on the 'registered land' area.[19] They show that the production response in agriculture when comparing

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[18] That is, in the livestock sectors cattle is considered as reproductive capital.

[19] Unregistered agricultural land does not qualify for the SFP.

domestic support elimination with domestic support elimination PLUS the implementation of the SFP, are the same. This is because in 'relative' terms, uniform land payments do not favour any particular agricultural activity. The authors claim that such a modelling choice is useful in that the payment is decoupled from production, coupled to the land factor, and yields useful estimates on the extent to which the SFP is recapitalised into the value of the land factor (i.e., through increased market prices of land). The modelling 'trick' here is that by applying a uniform subsidy rate on all land, the additional change in the sector specific land price for each sector 'j' is equal to the change in the aggregate land price in the CET land nest, which eliminates price effects. Consequently, the additional supply response when implementing the SFP uniform land payment is zero (i.e., no additional production or trade effects).

To model this policy, in the database three land values are required: the agents (farmer) value of land (P1LND); the market value of land exclusive of SFPs but inclusive of agenda 2000 coupled payments (P1LNDL); and the market value of land inclusive of *all* subsidy payments (P1LNDM). In the database, for each industry 'i', we therefore have three land value flows, which have the following update statements:

$$\begin{aligned} &\mathrm{V1LND(i)} = \mathrm{P1LND(i)} \times \mathrm{X1LND(i)} \\ &\mathrm{newV1LNDM(i)} = \mathrm{P1LNDL(i)} \times \mathrm{X1LND(i)} \qquad (13) \\ &\mathrm{V1LNDM(i)} = \mathrm{P1LNDM(i)} \times \mathrm{X1LND(i)} \end{aligned}$$

In the benchmark 2005 database, 'newV1LNDM(i)' and 'V1LND(i)' are equal across all 'i', implying no SFP. In the price transmission equation between P1LNDL(i) and P1LNDM(i) an exogenous (non indexed) subsidy wedge called 'GREENBOX' is inserted, which when swapped with the nominal value of SFP to be allocated, allows the modeller to insert a uniform land subsidy for all sectors 'i'. Simultaneously, it is possible to strip out coupled Agenda 2000 payments by swapping the exogenous land subsidy rate (indexed over 'i') between V1LNDM and newV1LNDM, with the value of subsidy wedge value for that industry 'i'.

Historical data on the SFP payments to Spain are taken from the Fondo Español de Garantia Agraria (FEGA), whilst coupled payments are stripped out year on year (in the dynamic simulation) based on historical data (up to 2010) and subsequently, assumptions based on agricultural policy developments. It should be noted that 'pillar 1' SFP are reduced by 5% (10%) in the Mid Term Review (Health Check) scenarios to reflect modulation to Pillar 2.

5.5 Set aside

Given that set-aside is mainly composed of an obligatory component[20] (subject to the small farm exemption), we treat this as an exogenous policy variable. Thus, we characterise the set-aside of cereals, oilseeds and protein (COP) crops sectors employing an exogenous productivity variable (a1lnd),[21] where exogenous negative (positive) shocks to the variable $a1lnd_i$ capture reductions (increases) in set aside land compared with the 2005 benchmark. Introducing the concept of an effective unit of land (denoted by superscript 'e') and a productive quantity of land the relevant price and quantity equations (in linear terms) are denoted as:

$$\begin{aligned} x1\ln d^{e}{}_{i} &= x1\ln d_i + a1land_i \\ p1\ln d^{e}{}_{i} &= p1\ln d_i - a1land_i \end{aligned} \qquad (14)$$

Thus, a positive exogenous *reduction* in 'a1lnd' by 10% implies that for every hectare used, only 0.9ha is effectively productive, which is equivalent to a set-aside of 10%. Moreover, since the effective unit of land has fallen, the unit cost necessary to produce the same amount of output will rise.[22] In the Hicksian demand equations for land, the components of effective demand and price of land are inserted as follows:

$$x1\ln d_i + a1land_i = x1va_i - \sigma_i\left[p1land_i - a1land_i - p1va_i\right] \qquad (15)$$

where p1va and x1va are the price and quantity indices of value added in the nest for each industry 'i'. These effective prices of land also enter into the zero profit function via the price index $p1va_i$. For the removal of set aside, a positive shock to a1lnd increases the productivity of land (reflected by the fact that more land is now in production). Notwithstanding, since set-aside land is usually marginal, we assume that the removal of 10% set aside (i.e., increase in land area by 10%) only warrants a productivity improvement of 5%

5.6 Export subsidy and quantity controls under the Uruguay Agreement and stock purchases

This section explains the modelling required to maintain the Uruguay Round (UR) export subsidy and quantity limits on extra-EU exports and the implementation of stock buying. The benchmark year for the database is 2005, which implies that as a 'developed country', Spain has completed its UR export subsidy and quantity commitments. Thus, in a

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[20] There is also a smaller voluntary component of set-aside.

[21] This is consistent with the approach employed in Bach et al., (2000).

[22] As a result, it is these effective prices are also included within the zero profit expression.

status quo scenario where the export subsidy is not eliminated, it is necessary to enforce the Spanish UR export commitments, whilst allowing for stock purchases.

Figure 12 presents a simplified partial equilibrium analysis of the UR export subsidy and quantity relationships. It is assumed that the domestic market price, P_m is fixed, which implies that the export subsidy expenditure function, in the lower part of the figure, is positive even at the free trade export quantity (Q_f). For the sake of argument, assume that the export quantity binding limit is Q^o, whilst the subsidy expenditure limit is E_1, then it is the quantity binding limit which is binding. Thus, the export quantity cannot exceed Q^o, whilst the export subsidy expenditure will only increase if the per unit export subsidy rate was exogenously increased (i.e., XS shifts to the left).

In terms of the model code, it is necessary to specify the following complementarity:

```
COMPLEMENTARITY
(Variable = V4TAX, Lower_Bound = 0, Upper_Bound = V4LIMIT)              (16)
E_EXPSUB (all,c,EXPSET)(all,s,XROW) X4(c,s) - X4LIMIT(c,s) ;
```

In policy terms, this equation states that there are two limits to adhere to for the variable V4TAX (i.e., the export subsidy). The lower bound is zero, and the upper bound is the UR agreed limit set by the variable, V4LIMIT. The variable X4 is the export quantity, whilst X4LIMIT is the UR agreed export quantity constraint. Thus, if the export subsidy equals V4LIMIT, then the expression X4 – X4LIMIT must be less than zero (i.e., export quantity is below or equal to the ceiling limit). If the export subsidy is greater than zero but less than V4LIMIT, then the expression X4 – X4LIMIT must be zero (i.e., export quantity is equal to the ceiling limit - binding). If the export subsidy is zero, then the export quantity is greater than or equal to the UR limit.[23] Thus, if the export subsidy expenditure is binding, the quantity constraint is less than or equal to the binding limit. If the quantity constraint is binding, then the export subsidy is less than or equal to the limit.[24]

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[23] In practise, this state never occurs, since if we wanted to eliminate the export subsidy, we would simply apply an exogenous shock.

[24] The downside of this treatment is that it doesn't deal with a scenario when neither is binding. One, or the other, or both must be binding. Consequently, when focusing on the UR export subsidy constraints it is useful to run a prior simulation in order to ascertain whether any specific export commodities (on non´-EU routes) do not violate either restriction. If this is so, then a complementarity is not modelled for that commodity.

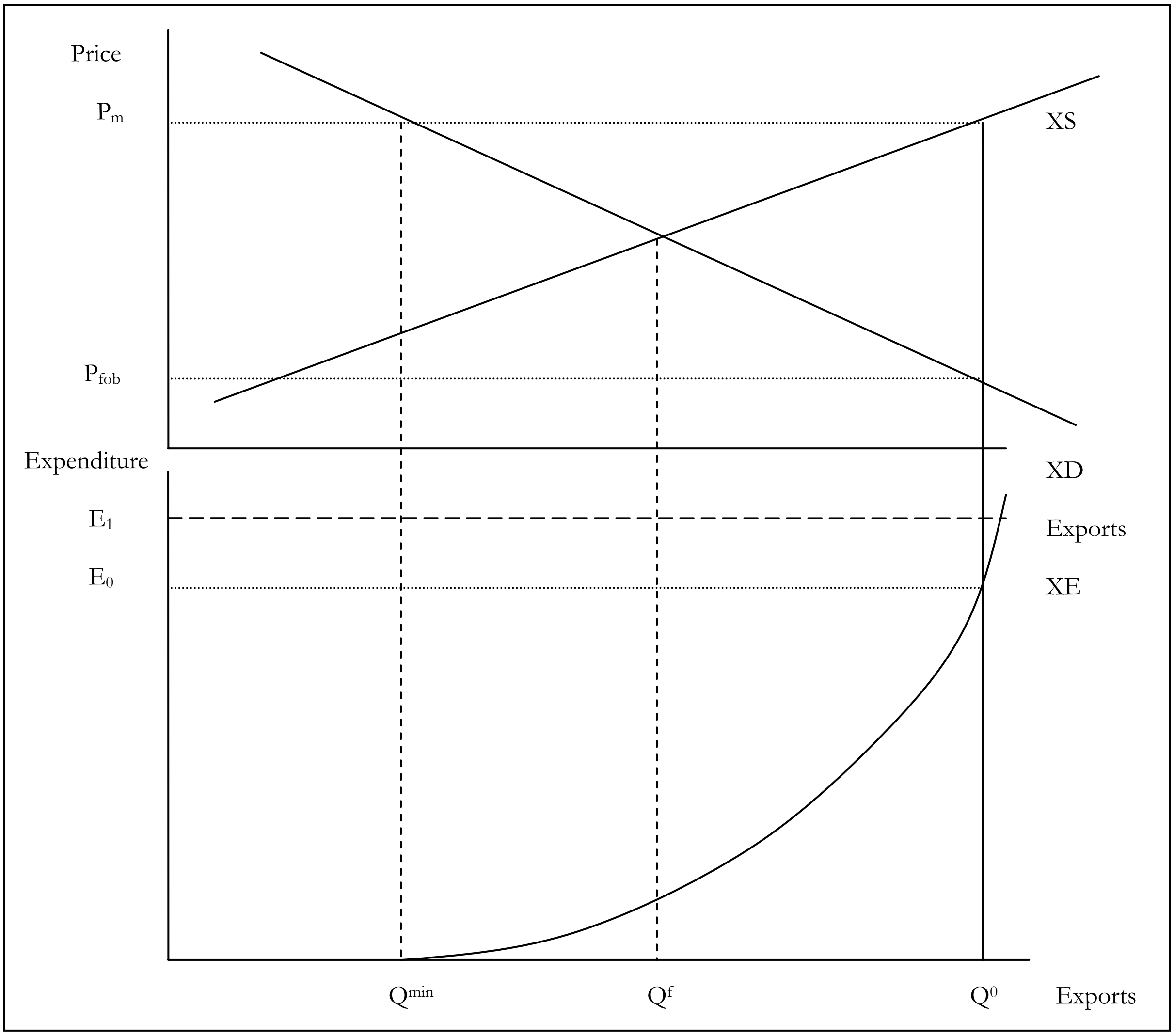

Figure 12: UR Export Constraints for EU countries:
A Partial Equilibrium Analysis

In the ORANI-DYN model, the export subsidy rate (expt4) for non-EU exports is endogenised to pair off with the COMPLEMENTARITY equation and ensure closure. Note that the per unit subsidy is the difference between the 'basic' (i.e., domestic pre-subsidy) non-EU export price (pe) and the non-EU export f.o.b. world price (p4). In the model, the domestic commodity price (which is the internal market price), p0com, is a weighted average of the domestic sales price (p0dom) and the composite basic export price (pe_c).

There are two components to intervention prices: The first is to model the stock purchase trigger; the second is to model the reduction in effective protection to EU producers from the price reduction. On the first modelling issue, stock purchases are captured employing a separate COMPLEMENTARITY expression:

```
COMPLEMENTARITY
(Variable = STOCKS, Lower_Bound = LOWER, Upper_Bound = CEILING)    (17)
E_c_STOCKS (all,c,EXPSET)(ALL,s,DOMES)  P0COM(c) - PINT(c,s) ;
```

where the intervention price is an (exogenous) policy controlled variable, PINT. Thus, given knowledge of the ratio between the intervention price and the domestic Spanish price, one knows how far P0COM has to fall in order to trigger stock purchases. For example, if P0COM has a normalised value of 1, and we know that the ratio P0COM / PINT is 1.002, PINT is therefore calculated as 0.998. In the equation, if P0COM – PINT is greater than zero (i.e., P0COM > PINT), then STOCKS are zero. If P0COM – PINT equals zero, then stocks are triggered (i.e., STOCKS > zero). If stocks reach their upper bound (denoted by CEILING which equals 2% of total commodity supply), then P0COM – PINT is less than zero. The intuition in the final state is that stock purchases are not indefinite and will ultimately cease, thereby allowing the market price to fall below the intervention price.

To model intervention price reductions, the variable PINT is exogenously reduced by the relevant percentage to lower the stock triggering point. Moreover, to capture the reduction in effective protection afforded to Spainish producers from intervention price reductions, in the non-EU (lower Armington) import demand function, (exogenous) import prices are reduced. In addition, with reference to the export demand function below:

$$X_{c,s} = F_{c,s}\left[\frac{P_{c,s}}{WP_{c,s}.ER}\right]^{-\sigma} \qquad (18)$$

$X_{c,s}$ is export demand by destination 's' for commodity 'c', $P_{c,s}$ is the corresponding price in local currency units, WP is the world price (modelled as an exogenous price shifter), ER is the exogenous exchange rate, $F_{c,s}$ is an exogenous quantity shifter variable and σ is the export demand elasticity. Thus, on the export side, intervention price reductions are imposed by a negative shock on F (the magnitude of this shock is calculated by knowledge of the intervention price fall and the export demand elasticity). Similarly, world price changes can be exogenously imposed on the variable WP. Thus, a reduction in world prices for commodity 'c' to destination 's=non-EU' will, ceteris paribus, reduce demand for Spanish extra-EU exports of commodity 'c'.

5.7 The farm household

As a useful summary statistic to policy makers, further equations are inserted into the ORANI-DYN model code to calculate farming income changes. More specifically, farm household income is simply the addition of 'on-farm' (factor payments) and 'off-farm' (i.e. support payments and modulation) income sources. Thus:

(i) The rental return to 'agricultural' capital and land, the nominal wage on agricultural labour (all valued at factor cost) plus the quota rent from milk and sugar sectors.

(ii) Support payments based on the 'subsidy wedges' on capital (headage payments, investment aids etc), land (set–aside, area compensation etc.), intermediate inputs (seed payments, young farmers allowance, LFA payments etc.), production (direct payments on olive oil, wine etc.) and export subsidies.

(iii) Modulation payments calculated on a rising scale percentage of the single farm payment.

In the model, separate accounting equations are introduced to separate nominal on-farm and nominal off-farm income, which are then added together to form a total farm income aggregate. In the model, real farm income changes are calculated by deflating the nominal equations by the consumer price index. These equations are reported merely as summary statistics and therefore have no impact on the model solution.

6. Recursive Dynamics: Capital accumulation and investment allocation

For the construction of medium to long run forecasts, an important drawback of the comparative static variant of ORANI is the lack of detail in determining the relationship between investment allocation and capital accumulation over time. The recursive dynamic extension of the ORANI model described here closely follows in spirit the characterisation within the MONASH model (Dixon and Rimmer, 1998). The key characteristic of a recursive dynamic model is that each solution is solved period-by-period (as opposed to an intertemporal dynamic model where results are computed simultaneously for all periods). Thus, the updated database for the current period becomes the starting point for the next period.

Employing 'stock-flow' concepts, the underlying relationship between new investment (flow) and the existing capital stock is given by the capital accumulation function (dropping industry subscripts for simplicity):

$$K_t = I_{t-1} - D \times K_{t-1} \qquad (19)$$

where changes in the capital stock in the current period (t) are a function of changes in new investment in the previous period (I_{t-1}) less the depreciation rate (D) on existing capital stock in the previous period (K_{t-1}). In determining how new investment is allocated across industries, the underlying hypothesis is that investors in industry 'i' compare expectations of the rate of return with historical or 'normal' rates of return. As a proxy for risk perception, investors will only be willing to supply investment to industry 'i' above some normal rate of growth if they have an expectation that they will be compensated by a rate of return which exceeds the normal rate of return. In the model, the normal rate of return is given as the ratio of rent payments on the stock of capital (P_k), divided by the per unit cost of new capital good construction (i.e., investment), denoted by Π:

$$R = P_k / \Pi \tag{20}$$

The underlying behavioural assumption of investors is that of adaptive expectations. Thus, with a single year time lag, the level of investment which accrues in industry 'i' is a positive function of the ratio between investors expectations of rate of return (E) and actual rate of return (R).[25] More specifically, (dropping industry subscripts for simplicity) it is postulated that the expected rate of return in the current period (E_t) is some weighted average of the expected rate of return in the last period (E_{t-1}) and the normal rate of return in the current period (R_t). In levels (as opposed to linear) terms:

$$E_t = (1-x)E_{t-1} + xR_t \quad 0 \leq x \leq 1 \tag{21}$$

Since expected rates may diverge from the actual or normal rates of return, there are errors in expectations. Indeed, the degree of convergence between expected rates of return and normal rates of return is a positive function of the parameter x. That is, if 'x' assumes a value of zero, the expected rate will always be a function of the previous periods expectations, which implies zero convergence. Conversely, at the other extreme where 'x' assumes a value of 1, convergence occurs in a single period. This would imply that the rate of growth of capital in each industry is always equal to the historical growth rate (see discussion on equation 23 below). A value of 'x' between 0 and 1, implies that investors' expectations slowly converge on a long run normal rate of return, where the number of periods of convergence are less, the larger is 'x'. In our model, we hypothesise that investors are cautious employing a higher weighting on the previous years expected rate of return (i.e., $x < 0.5$). Too low a parameter value can result in cycling where overly pessimistic expectations of the real rate in period N can cause capital scarcity and overly high actual rates of return which in period N+1 results in capital abundance and unrealistically large falls in actual rates of return. By running simulations for robustness, we find that 0.33 provides a satisfactory convergence over the solution time horizon.

The next task is to describe the relationship between expected and actual rates of return, with rates of capital growth. The rate of capital growth is a function of the ratio of new investment (new 'flows' of capital goods construction) to the existing 'stock' of capital:

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[25] See Part I, section 18 on the initial period values chosen for rates of return and the capital stock.

$$G_t = I/K \quad (22)$$

Relating changes in the expected rate of return (E_t) to capital growth (G_t) in period ‘t’, it is implicitly assumed that the latter is a positive function of the former. Moreover, it is important that when expected rates exceed (are below) normal rates of return, the rate of capital growth is above (below) the historical growth rate of capital. These relationships are captured by the logistic capital accumulation function (dropping industry subscripts):

$$G_t = Q.G_{trend} M_t^{\beta} /(Q - 1 + M_t^{\beta}) \quad (23)$$

where G_{trend} is the historical rate of capital growth; M_t is the ratio of expected to normal rates of return in period t; the parameter Q is the ceiling limit multiple of the capital growth rate to the historical growth rate of capital, and β is the elasticity of G_t with respect to changes in M_t. Thus, if the expected rate of return (E_t) is equal to the normal rate (R_t), then M_t is equal to 1, and therefore via changes in the capital stock in equation (19) the rate of capital growth in period ‘t’ is equal to the historical or trend rate ($G_t = G_{trend}$). Similarly, if M_t is zero then G_t is also zero, implying that for capital growth to occur, the expected rate of return must be positive (i.e., with zero expectations, individuals will not invest). Finally, if M_t assumes a ‘large’ value, then G_t closely approximates the ceiling limit multiplied by the trend rate. In other words, with E_t values exceeding R_t beyond a certain limit, the rate of growth of capital cannot exceed a certain threshold. Examining different values of Q, G_{trend} and β, it is possible to track the relationship between M_t and G_t.

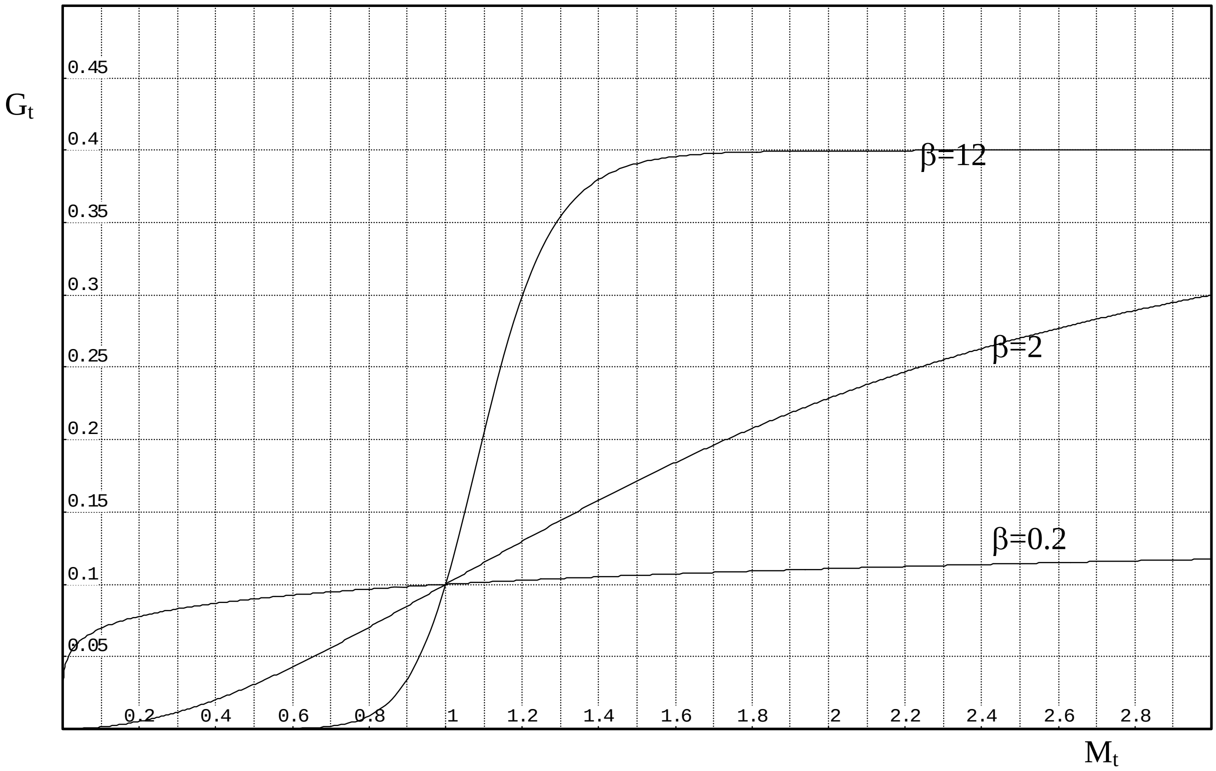

Figure 13: Investment sensitivity to rates of return. (Source: Horridge, 2002)

Thus, given that the trend rate of growth is 0.1 (10%), then when M_t is equal to 1, all three curves correspond to a G_t value of 0.1. The larger is β, the more sensitive is the responsiveness of capital growth where with an extreme value of β=12 and an M_t value of 1.8, implies that G_t is at its ceiling limit (i.e., G_{max}) of four times (parameter Q=4) the trend growth rate of 0.1. With a β value of 0.2, capital growth is relatively unresponsive to changes in M_t (in other words, the changes in the expected rate of return with respect to normal rates of return, must be considerably larger to realise substantial capital growth). For the purposes of this model, we assume that investment allocation is relatively sensitive to changes in M_t (i.e., in the post 2005 era, we assume that investors are on the look out for even small potential improvements in their expected rate of return in times of economic uncertainty). Consequently, we assume a β value of 5. In checking the robustness of the model (in light of the number of modelling features), we discover (not surprisingly) that reduced investment volatility produces greater accuracy and robustness. More specifically, a Q value of 4 is employed (i.e., capital growth cannot exceed more than four times the historical rate). In determining G_{trend}, data between the period 1996 to 2007 reveals that capital stocks grew on average by 10.6% (INE, 2010). However, this data reflects a period of unprecedented growth and prosperity in Spain, whilst the outlook in the post 2005 period will be characterised by lower growth rates of capital (on a larger base). Consequently, we assume a G_{trend} value of 0.05 (5%).

An additional extension of this dynamic investment mechanism allows the modeller to characterise the observed tendency for industries to hold excess capital capacity in times of recession. Dixon and Rimmer (2010)[26] note that the typical (and simplistic) 'full-capacity' utilisation assumption of neoclassical CGE models (i.e., no idling of capital) implies an unrealistically high reduction in rental rates when demand falls and a new market clearing equilibrium is found. With such increases in rental rates, an undesirable consequence of a recession scenario is that aggregate prices fall excessively with a resulting surge in exports. In contrast, by modelling excess capacity, there is sticky adjustment downwards in rental rates as capital stocks are left unemployed.[27]

To model this, we begin with the following equation (adapted from Dixon and Rimmer (*op. cit.*)), which compares the rental rates on capital stocks in the baseline scenario (no recession) with the policy scenario (recession) (drop industry subscripts for simplicity).

$$\left[\frac{P_{kt}^{P}}{P_{kt}^{B}}-1\right]=\left[\frac{P_{kt-1}^{P}}{P_{kt-1}^{B}}-1\right]+\alpha.\left[U-1\right]+SLACK \qquad (24)$$

Thus, the capital rental rate in the policy (crisis) scenario in period t, (P_{kt}^{P}), relative to the rental rate in the baseline (no crisis) scenario in period t, (P_{kt}^{B}), is a positive function of the corresponding ratio in period t-1, a negative function of excess capital capacity (U) measured by the ratio of capital in use in the policy to capital in existence (employing the baseline estimate); and a slack variable. As a first step, both baseline and policy experiments are run with the slack variable endogenised, the ratio U exogenised:

$$U = KU^{P} / KE^{B} \qquad (25)$$

such that that KU^{P} and KE^{B} grow at the same rate (i.e., U=1),[28] and the ratios of rental prices in current and previous periods exogenised. These three conditions ensure that equation (24) is switched off. Having conducted our initial experiment, the difference in the rate of capital uptake in the policy with respect to the baseline is recorded, for the crisis years only (where KU^{P} is well below KU^{B}). Subsequently, the policy is rerun where U is now shocked to mimic the reduced uptake of KU^{P} to KE^{B} (i.e., U < 1), whilst the 'slack' variable (now exogenous) in equation (24) is swapped with the capital rental ratio (now

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[26] Dixon and Rimmer employ a single country dynamic CGE model (USAGE) to examine the impacts of the Obama stimulus package on the US economy.

[27] It should be noted that we are not physically modelling stock accumulation of capital, but rather representing excess capital capacity via the sticky downward nature of capital rents

[28] The definition of 'capital in existence' employs the Australian technique of forecasting, where the forecast (or baseline in our jargon) allows the modeller to anchor the results of the policy scenario to those in the absence of the shock (see Dixon and Rimmer, 2010). In their baseline, Dixon and Rimmer assume that the variable KE is made exogenous instead of U, but the principle is the same.

endogenous) to allow policy rental rates to diverge with respect to baseline rental rates, the extent of which is determined by the size of the positive parameter α. Thus, in equation (24) it is assumed that the deviation of the rental rate in the policy from its baseline is proportional (controlled by the parameter α) to the deviation of capital usage in the policy (KU) from its baseline level (KE^B). This is made clearer in Figure 14.

Figure 14 (Dixon and Rimmer, 2010) presents the supply and demand for capital in our economy. In the baseline run, demand for capital is D_B, which at full capital usage (KE) gives a rental rate of R_B. If we did not take account of idle capital stocks in the policy scenario, the reduction in demand would result in a fall in capital demand to D_P^1, with a rental rate of R_p^*. When accounting for excess stocks of capital (KU < KE), this rental fall is 'corrected' to R_P in the policy scenario. The parameter α in equation (24) controls the proportionality between the deviation of the rental rate from its baseline level and the deviation of capital employed from the (full capacity) baseline level. With reference to equation (24), the larger is α the more inelastic is the capital supply function below KE such that the deviation in policy and baseline rents under excess capacity increases for a given level of excess capacity.

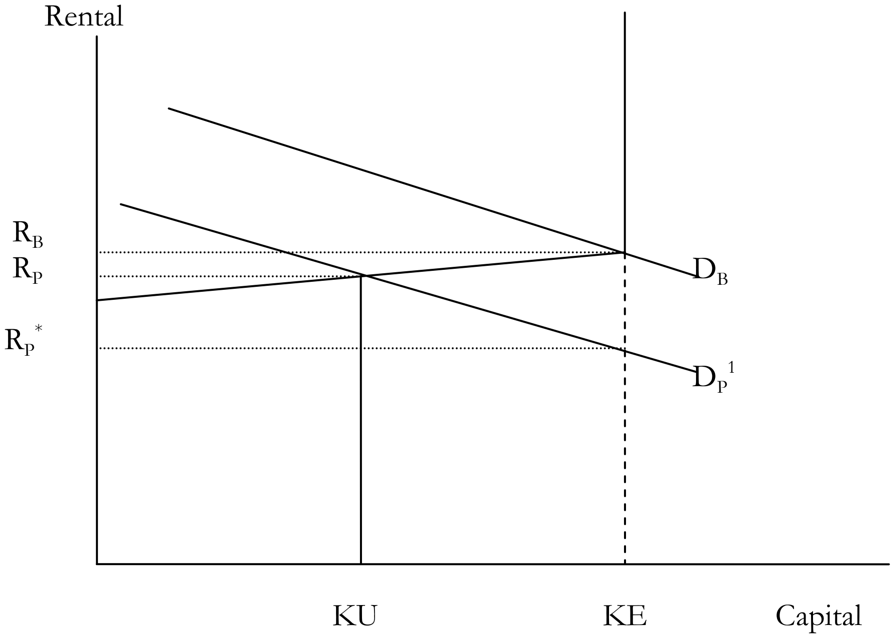

Figure 14: Supply and Demand for capital

It is not possible to calibrate the paramater α to time series data since we do not know what rental rates would have been had the crisis never taken place. Consequently, one may assume that capital stock reductions from the policy scenario are a short run phenomenon (i.e., excess stocks are run down reasonably quickly as business conditions pick up) rather than a long term structural problem. This implies a more elastic capital supply function below KE. Consequently, this criteria favours the choice of a smaller α parameter value. On running repeated robustness tests a value of 0.5 was elected.

Having determined the impact of excess capacity on capital rental rates, it is also necessary to modify the impacts of excess capacity on expected rates of return and capital accumulation. Concentrating first on expected rates of return, we follow Dixon and Rimmer (*op. cit.*), by modifying equation (21) to:

$$E_t = U.\left[(1-x)E_{t-1} + xR_t\right] - (1-U).D \qquad 0 \leq x \leq 1 \qquad (26)$$

Thus, in the crisis years only, the rate of return is adjusted downwards owing to the existence of capital which is held idle (i.e., excess supply of capital). More specifically, the rule for investors' expectations remains unchanged for that proportion of capital which is in usage (U < 1), whilst the rate of return on idle capital (1-U) is a negative function of the depreciation rate (D). In other words, capital which is not in use accrues no return and deteriorates at the rate of depreciation (Dixon and Rimmer, (*op. cit.*)). In the remaining years where no excess stocks of capital are held, equation (26) collapses to equation (21) and equation (24) is effectively switched off via a change in the model closure.

Finally, the impact of excess capital capacity is introduced into the capital growth equation:

$$G_t = Q.G_{trend} M_t^{\beta} U / (Q - 1 + M_t^{\beta}) \qquad (27)$$

where the impacts of new investment on capital growth rates are moderated when there is excess capital stock in the crisis years (i.e., U < 1).

7. Adjustment of (sticky) real wages to employment

In a recursive year-on-year recursive dynamic framework, it is useful to have a mechanism for capturing the medium term relationship between changes in wage growth and employment. In particular, when modelling labour market rigidities (particularly pertinent in the case of Spain), it is useful to be able to calibrate the relationship between changes in real wages and employment levels, if possible via the usage of historical secondary data. In the dynamic version of ORANI, the approach is to gauge the relationship between changes in the employment level in a given period 't' (L_t) and changes in the long run 'trend' level of employment (T_t), with changes in the real wage.[29] In our typical dynamic ORANI closure, employment in each period is controlled exogenously,[30]

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[29] The trend level could be thought of as as the level of employment corresponding to NAIRU

[30] It should be noted that by controlling employment rates year on year, we are implicitly taking account of changes in unemployment.

whilst changes in the trend rate are determined by population changes (exogenous and typically shocked – see section 9) and the participation rate of the workforce (exogenous). From the equation below:

$$\Delta W / W_t = \gamma[(L_t / T_t) - 1] + \gamma\Delta(L / T) \tag{28}$$

If end-of-period employment growth exceeds the trend level by x%, then real wages will rise by γ.x%. Over time, rising real wages are unsustainable if they exceed productivity/growth improvements. This implies that real wages must fall, with resulting in a fall in L_t toward the long run employment trend. Eventually, the long run real wage (i.e., where ΔW = 0) corresponds to a situation where $L_t = T_t$. To capture the sticky downward nature of real wages in Spain, it is assumed that L_t starts equal to T_t in the intial period; historical data on real wages and employment from INE, (2010) and population changes from IMF (2007, 2010) are implemented, whilst γ is calibrated such that exogenous changes in L_t produce real wage changes which are close to historical data.

In addition to the relationship between macro real wages and macro employment changes, linear labour supply functions are incorporated into the model in order to characterise the supply responsiveness of different labour occupations to changes in the real wage. To support this modelling feature, central tendency estimates of labour supply elasticities for Spain (0.3) are taken from Fernándes-Val (2003). Highly skilled labour types are assumed to have half the supply elasticity, whilst unskilled labour is assumed to have ten times the elasticity value. With a very large unskilled labour surplus in Spain (owing to high immigration and the contraction of the construction industry), the supply of labour is seen as very elastic.

8. Fiscal extension and disposable household incomes

The standard ORANI-G model makes no explicit link between household incomes and household spending. This is because the Spanish IO tables do not include data on direct income taxes or government transfers – that is we know the value of household spending, but not of disposable income. In addition, it is useful to include a fiscal policy module by government in order to calculate a government budget which can be exogenously adjusted to changes in macro conditions via a closure swap, or simply employed to present endogenous summary results. To capture these relationships, this section specifies:

A *proportional* link between total government expenditures and government consumption of goods/services and welfare expenditures

A *proportional* link between total government revenues and current government direct, and indirect income taxes and social security revenues

A *proportional* link between household disposable income and primary factor returns less direct income taxes

Note that the word 'proportional' is employed since the the value flows underlying the household module (i.e., household incomes and expenditures) are not equal.

To encompass the fiscal activities of government and record the impacts of direct taxation of private household purchases, we follow the fiscal extension employed in Wittwer (1999). In its standard format, the ORANI-G model has a partial image of the activities of government. More specifically, current government expenditures on goods and services (V5TOT) as well as indirect taxes (V0TAX_CSI) are recorded within the database. Notwithstanding, there is no account of public expenditure on capital (V2TOT_G), whilst information on the net transfers which exist between government and private households is also omitted.

To capture the fiscal role of government, we have two equations recording government expenditures (GOVEXP) and government revenues (GOVREV):

$$\begin{aligned} &GOVEXP = \\ &V5TOT + V2TOT\_G + TRANSFER("spend") \end{aligned} \tag{29}$$

$$\begin{aligned} &GOVREV = \\ &INDTAX + V0HHTAX + TRANSFER("rev") \end{aligned} \tag{30}$$

where V0HHTAX is direct income taxes on production factors owned by households; TRANSFER("spend") is public expenditure on social welfare payments, capital payments and other non financial assets; and TRANSFER("rev") which is public receipts on social security payments and rents on capital owned. In Table 2, the public accounts for Spain 2005 are as follows:

€ millions	**Income**	**Expenditure**	**Balance**
V5TOT	-	160,250	-160,250
V2TOT_G	-	32,354	-32,354
TRANSFER	142,151	155,035	-12,884
INDTAX	110,963	-	110,963
V0HHTAX	105,021	-	105,021
TOTAL	358,135	347,639	10,496

Table 2: Public sector accounts in Spain for 2005

Source: Spanish national accounts for 2005 and Spanish IO Tables, INE (2011)

The data headers are taken from the annex tables of the Spanish national accounts (V2TOT_G, TRANSFER, V0HHTAX) as well as the Spanish IO table for 2005 (V5TOT, INDTAX).[31] To update these value flows, government transfers (TRANSFER) are assumed to change at a constant rate with the value of Spanish GDP and an exogenous shift variable (which allows the user to implement greater/lower public spending as a proportion of GDP). Public expenditure on capital changes in line with economy-wide new capital expenditure (i.e., it is assumed that public expenditure on capital remains a constant share of total capital investment) and an exogenous shifter variable (employed to change the ratio of public to total expenditures on investment). Aggregate household income taxes change in line with changes in aggregate household disposable income (see below) and an exogenous shift variable (the shifter can be employed to increase or reduce direct income taxes).

The 'change' in the government budget is calculated as:

$$100.delbudget = GOVEXP.w0gov\_g - GOVREV.w0gov\_t \qquad (31)$$

where w0gov_g and w0gov_t are the percentage changes in government expenditures (equation 29) and revenues (equation 30), respectively. Conceivably, the government budget could be made exogenous and shocked year on year by swapping with the shifter on current government expenditures (f5tot).[32]

As an additional extension, our Spanish model also includes a relationship between aggregate household gross income (calculated based on the returns from the factors of production) less direct taxes and social security payments which gives net disposable incomes. The prefereed approach would be to link disposable incomes and expenditures to each of the eight households in the datrabase. Unfortunately, owing to a lack of secondary data on the proportion of factor earning or social security payments which accrue to each household segment, we employ the next best alternative of proportionally linking 'aggregate' household disposable income with 'aggregate household' expenditures.

Aggregate household nominal take-home income may be calculated by the expression:

$$\begin{aligned} V0HHINC = {} & \sum_{i=IND} LAND + \sum_{i=IND} CAPITAL + \sum_{i=IND} LABOUR + \\ & TRANSFER("spend") - TRANSFER("rev") - V0HHTAX \end{aligned} \qquad (32)$$

Aggregate household income accrues on all returns to the factors of production (i.e., land €9,208m), capital (€364,866m) plus 'gross' salaries (€430,734)) and welfare payments

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[31] In the national accounts, indirect tax revenues are €110,963 million, whilst government expenditures on subsidies are €9,151 million. This gives a net indirect tax figure of €101,812 million. This is slightly different from the 'net' indirect tax figure recorded in the Spanish IO Table for 2005 (€99,502).

[32] The reader is encouraged to consult Horridge (2003) to understand the role of f5tot in the ORANI-G model.

(TRANSFER("spend")), less social security payments and rents on publically owned buildings (TRANSFER("rev")) and direct income taxes on salaries, estates etc (V0HHTAX). Summing over these concepts gives the value €712,671 million. In the standard ORANI-G model, the percentage change in aggregate household expenditure (w3tot_h) is linked to changes in the value of Spanish GDP (w0gdpinc). In our modified model, percentage changes in aggregate household expenditure and direct income taxes (w0hhtax) are now proportionally linked to aggregate household disposable income (w0hhinc) via the linear equations:

$$\begin{aligned} w3tot\_h &= f3tot + w0hhinc \\ w0hhtax &= w0hhinc + finctax \end{aligned} \tag{33}$$

where w0hhtax is used to update V0HHTAX, w0hhinc is employed to update V0HHINC and f3tot and finctax are exogenous shifter variables. The latter can be employed to manipulate (uniform) direct taxes rates, which also has feedback effects on the government budget.

9. Closure and shocks in dynamic ORANI-DYN

As is typical in the CGE model approach, the number of variables (n) will exceed the number of equations (m), which requires (n-m) exogenous variables. From a mathematical perspective, the exogenous-endogenous split must ensure that the endogenous coefficient matrix is invertible. It is, however, the realm of economics to which we must turn in order to guide our choice of appropriate closure. This may be guided by specific considerations:

Firstly, how available are the data? Economics has relatively little to say about non price determinants (i.e., taste, productivity). Consequently, such variables are typically maintained as exogenous over time. If 'acceptable' proxy data are available, then exogenous shocks may be applied within a baseline scenario (see later discussion).

Secondly, what is the maintained hypothesis for the macro economy. It may be that the modeller wishes to focus on the short run impacts. In this case, a closure where real wages are fixed (controlled by unions) under the 'sticky wages' hypothesis is more appropriate. With a closure change, a long run labour market scenario could imply fully flexible real wages (i.e., labour supply fixed and real wages fully flexible).

Thirdly, there is the issue of what the focus of the simulation is. For example, it may be that the impact of milk quotas is of paramount importance – thus, the milk quota must be exogenised, whilst rent should be allowed to adjust endogenously. Alternatively, if an import quantity is exogenised, the corresponding price imported will need to be endogenous (to capture, say an import quota). Likewise, with a price fixing closure (exogenous price), the quantity should be allowed to adjust. If both price and quantity are exogenised in a given market, we must allow the non own-price determinants to change endogenously (i.e., allow the curve to shift) – which implies the need to endogenise taste (demand) or technology (supply) variables.

Within a system of 'n' equations, an examination of period-by-period equilibria requires a careful and consistent selection of exogenous shocks. In the case of our ORANI baseline and policy scenarios, taste change data on red/white meat taste changes based on OECD (2008) forecast data measured in per capita consumption (kg per head). Productivity shocks for crop, ruminants and non ruminants sectors are based on estimates of total factor productivity change in Ludena el al. (2006), whilst those for the non-agricultural sector employ shocks found in Jensen et al. (2003).

Given the small country assumption, it is assumed that the world price in import and export demand functions are exogenous. Whilst year-on-year world price data at the level of tariff line aggregation is not available for all agricultural and non agricultural sectors, we can at very least control for changes in fossil fuel and biofuel world prices. For fossil fuels, data from the United States Energy Information Administration (2010) is employed which provides data on coal, oil and gas with additional projections up to 2035. This data accounts for the crisis, especially in 2009, where crude energy prices dropped dramatically owing to the contraction in the US economy. For a non-crisis baseline, historical data is employed up to 2008, whilst from 2009, it is assumed that world prices rise 2% per annum (as assumed in OECD projections).

For biofuels, OECD (2009, 2010) provides useful year on year world prices data with forecasts for both non-crisis and economic crisis scenarios. The change in Spanish biodiesel and bioethanol activity is captured employing these year-on-year shocks, whilst further developments in these markets are endogenously determined via the chosen elasticities in the energy substitution possibilities nest.

In addition, EU and non-EU inflation changes can be implemented via a new world price variable indexed over bilateral route only. In this way, price shocks will impact uniformly across all commodities in the import and export demand functions. Data for EU and world inflation can be obtained from IMF (2008, 2010).

In ORANI-DYN, the total land endowments in Spain changes endogenously, employing an econometrically estimated land supply function (see section 5.2). In the labour markets (see section 7), the (sticky) real wage is calibrated to a wage elasticity parameter with exogenous changes in employment. Moreover, different supply elasticity functions characterise different degrees of labour rigidity (particularly pertinent for the Spanish labour market) across different labour occupations (more highly skilled implies lower supply elasticity). The total capital endowment is endogenous and adjusts in each period 't' (capital accumulation) to exogenous changes in net investment in period 't-1'.

In terms of agricultural product markets, subject to policy induced reductions in intervention prices, negative shocks are imposed on non-EU sourced import prices and a downward (negative) shock in the export demand shifter variable, whilst the quota quantity limit is exogenous (allowing binding/non-binding status) and can be increased when examining gradual phasing out of quota. The quota rent adjust endogenously. Year on year proposed reductions in agricultural land-, capital-, production- and intermediate input subsidies simulate the gradual removal of direct (agenda 2000) support, which are reconstituted within the uniform land subsidy payment which characterises the SFP (see section 5.4).

Turning to our macro variables, Spanish population (which affect the long run natural rate of employment) shocks are taken from IMF (2007) for non financial-crisis scenarios and IMF (2010) for crisis scenarios. In our Keynesian aggregate demand equation, private consumption (C), public consumption (G), investment (I) and aggregate exports (X) are targeted exogenously, as well as real GDP. Consequently, aggregate imports adjust endogenously to meet targeted changes in GDP, whilst the (numeraire) exchange rate is held fixed.

A macro-wide productivity variable is swapped with real macro growth to allow the modeller to target GDP growth over the time horizon of the baseline. Growth data are available for a non-crisis scenario (IMF, 2008 – this report had not foreseen the crisis) to 2010. In the absence of any reliable data estimates from recognised agencies/public bodies,[33] from 2011 to 2015, non-crisis growth estimates are assumed to equal that of the 2010 level (3.29%). This rate of growth is a little below the average growth rate over the period 1995-2007 (3.8%), although given that the Spanish economy has grown, it is seen as desirable that a perceived 'stable' growth rate in the 2011-2015 component of the baseline be relatively smaller. The period 1995 to 2007 is seen as a useful benchmark for projecting non-crisis conditions since it represents a period of uninterrupted stability and growth in Spain.

When applying year on year shocks, the remaining components of the aggregate demand function (less imports) are also held exogenous and shocked employing both historical and projections data. The suite of ORANI/MONASH models also commonly employ 'shifter' (scale) variables to enable swaps with strategic policy variables of interest. Thus, real private consumption is swapped with a macro taste shifter variable which impacts uniformly across all LES commodity demand functions. In the case of aggregate employment, an exogenous shifter on the real wage (an additional exogenous scalar on the right hand side of equation (25)) can be swapped with aggregate employment, such that real wages adjust to target levels of employment.

A similar modelling 'trick' is employed for investment, where a macro wide normal rate of return is inserted into the variable 'M' in the industry capital accumulation equation (24). Swapping with real capital growth, upward/downward shifts in this rate of return

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[33] Some Spanish sources were found, but we were not confident enough of using them.

impact on the variable 'M' (ratio of expected to normal rate of return) which consequently affects capital growth rates by industry ('G') such that the macro investment target variable is met. This also has implications on the rental rate of capital and the unit cost of capital construction. In each of the export demand functions, there is an exogenous macro demand shifter, and an equivalent demand shifter indexed by commodity. Swapping the macro shift variable with the aggregate value of exports, it is possible to target aggregate exports via a unfirm downward/upward shift across all export demand functions (indexed by commodity 'c'). Finally, in ORANI the 'default' setting is to link changes in public expenditure to private expenditure changes via an exogenous shift variable. By endogenising the shift variable and swapping with public expenditure, the modeller may apply policy shocks to government spending.

To shock each of the exogenous components of the Keynesian consumption function (i.e., C, I, G and X), for a non crisis scenario, historical data is available up to 2007 employing data from Eurostat (2010). Further reliable data for Spain across each of these components of aggregate demand is not available. Consequently, we employ time series data on C, I, G, and X between 1995 to 2007 as well as employment data from Eurostat (2010), whilst aggregate imports must adjust endogenously to meet the GDP targets. Employing time series data across C, I, G, X, employment and GDP, a representative correlation coefficient is calculated for each component of aggregate demand and employment, with respect to changes in GDP. Thus, given changes in the rate of GDP, via the correlation coefficient, we can calculated changes in C, I, G, X and employment. In this way, it is assumed that the composition of each element of GDP is a representative average of the 1995-2007 period.

In the case of a crisis scenario, estimates of Spanish GDP growth are available from IMF (2010), who provide long run annual projections up to 2015. Data on investment up to 2013 is taken from Eurostat (2010); whilst government expenditure (up to 2013); private expenditure (up to 2013); exports (up to 2011); and employment (up to 2013) are taken from the European Commission (2010). As noted above, to fill in the 'missing years' for these components of aggregate demand, a correlation coefficient with respect to changes in GDP is once again employed employing time series data up to 2013. Given changes in GDP, it is then possible to compute changes in the components of aggregate demand (i.e., C, I, G, X) and employment .

10. Conclusions

This document begins with a brief description of the standard ORANI-G model.[34] ORANI-G has received considerable recognition from modellers around the world since

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[34] As noted in the introduction, for a detailed understanding of CGE models and the ORANI-G model, consult Horridge (2003).

the data base and model structure are 'relatively' easily tailored to a set of published input-output accounts. Consequently, national economy models employing the ORANI-G template have been developed for over 30 countries around the world. Continuing in this tradition, the current document discusses the ORANI-DYN model; a specially tailored extension of the ORANI-G model for the Spanish economy. Since the priority of the model is to aid policy analysis in the agro-food sectors, the modelling of agricultural primary factor, intermediate input and output markets has been improved considerably. Moreover, given the rising importance of biofuels and the implications for land usage, explicit modelling of energy demands in the private household and production nests provides flexibility in examining the impacts of fossil fuel price changes on biofuels production through substitution possibilities within the nests. In addition, ORANI-DYN provides additional final demands accounts for 'inbound' tourism and NGO activity; labour and capital transfer between agricultural and non-agricultural sectors is modelled as sluggish; whilst trade links are now subdivided between EU and non.-EU routes.

Finally, to more correctly characterise the workings of the Spanish economy, supply elasticities and wage elasticity parameters are emoployed to capture the structural rigidities in the labour market, whilst a recursive dynamic investment module has been added to capture the time dependant impacts of investor expectations on capital accumulation in Spain.

Whilst ORANI-DYN represents an important departure from the standard ORANI-G model, and at the same time a useful policy analysis tool for agro-food policy in Spain, it should still be considered as an ongoing 'work in progress'. Further developments are likely to be included in the future, including an improved treatment of land to account for irrigated and non irrigated land substitutability in crops sectors, whilst also accounting for the importance of water demand in irrigated land. Further model developments centre on the macro/micro underpinnings of the model (e.g., imperfect competition, continued improvements in the search for relevant elasticity values), whilst some form of CAP budget would have implications for net injections/withdrawals from the Spanish economy and consequently the balance of trade sub-balance.

Notwithstanding, with its detailed multi-commodity/industry database, ORANI-DYN is the first agro-food CGE model for Spain and constitutes a useful platform for analysing in detail the impacts of forseeable and non foreseeable shocks in the Spanish economy.

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Appendix A: Nesting

The choice of function under conditions of model calibration favours the use of simpler 'convenient functional forms'. The drawback, however, is that simpler functional forms greatly restrict the number of parameters within the function, which in turn inhibits the degree of flexibility when characterising producer/consumer behaviour.

A common response to this problem is to employ a *separable nested* (or hierarchical) structure, whereby an assumption is made about the partitioning of the elements of the underlying production/utility function into different groups and aggregations. Hence, the assumption of separability implies that constrained optimisation is undertaken in several stages. Nested structures then allow a greater number of elasticity parameters at each stage of the production/utility function. This increases the flexibility of the model, without burdening computational facility.

Separability and Aggregation

In order to undertake a two-stage nested optimisation procedure, two conditions must be met. First, to permit a partitioning of the inputs, Strotz (1957) devised the concept of weak separability. A precise definition of separability is given by Chambers (1988) who notes,

'separability hinges on how the marginal rate of technical substitution (MRTS) between two inputs responds to changes in another input' (pp.42).

To illustrate the relationship between separability and multi-stage optimisation, a theoretical example is employed. Assume a 3 factor (x_i i=1,2,3) production function which is of the form:[19]

$$Y = f(X, x_3) \qquad \text{(A.1)}$$

where input X is represented as an *aggregator function* consisting of inputs x_1 and x_2:

$$X = g(x_1, x_2) \qquad \text{(A.2)}$$

A schematic representation of this two-level nested structure is presented in figure 2.5.

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[19] This theory is equally applicable to the utility function in consumer theory.

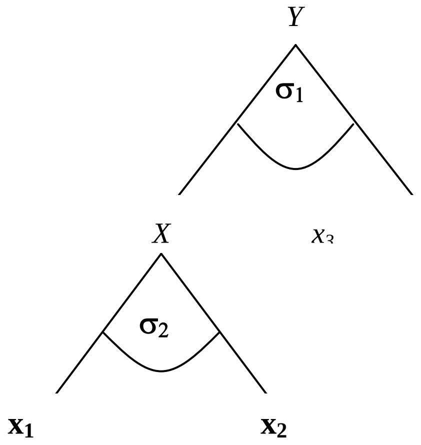

Figure A1: A Two-level nested production structure

It is assumed that the underlying production function (A.1) is weakly separable implying (using Chambers' (1988) notation):

$$\frac{\partial}{\partial x_3}\left(\frac{\partial X / \partial x_1}{\partial X / \partial x_2}\right)=0 \qquad \text{(A.3)}$$

In words, this expression states that the ratio of marginal products (MRTS) of inputs x1 and x2, belonging to the same input nest X, is not affected by changes in the level of input usage of x3 which is not in that nest. The family of convenient functions such as CD and CES exhibit weak seperability, where in the case of a two-level nested CD production example:

$$Y = AX_1^{\alpha} X_2^{\beta} \qquad \textit{and} \qquad X_1 = Ax_{11}^{\gamma} x_{21}^{\delta} \qquad \text{(A.4)}$$

The $\mathrm{MRTS}_{11,21}$ can be shown to be:

$$MRTS_{11,21} = \frac{MP_{11}}{MP_{21}} = \frac{\gamma}{\delta}\frac{x_{21}}{x_{11}} \qquad \text{(A.5)}$$

Clearly, changes in the level of X_2 in the upper CD nest, has no effect on the MRTS between inputs x_{11} and x_{21} in the lower nest. Mathematically:

$$\frac{\partial}{\partial X_2}\left(\frac{\gamma}{\delta}\frac{x_{21}}{x_{11}}\right)=0 \qquad \text{(A.6)}$$

The second condition is that the aggregator function (A.2) must be linear homogeneous with respect to each of its inputs. It can be shown that the output price composite of a linearly homogeneous function is linearly homogeneous in input prices. Thus, the aggregate quantity and price indices are equal to the sum of the prices and quantities of the inputs derived in each nest:

$$RX = \sum_{i=1}^{n} r_i x_i \qquad (A.7)$$

A basic property of linear homogeneous functions is that first order derivatives (i.e. marginal products/utilities) are homogeneous of degree zero. To demonstrate this property, take the case of a linearly homogeneous Cobb-Douglas production function. Hence for a two input production function, MP_1 is given as:

$$MP_1 = \frac{\partial Y}{\partial x_1} = \alpha A x_1^{\alpha-1} x_2^{\beta} \qquad (A.8)$$

Multiplying each of the inputs by a scalar, λ, yields:

$$\begin{aligned} MP_1 &= \frac{\partial Y}{\partial x_1} = \alpha A(\lambda x_1)^{\alpha-1}(\lambda x_2)^{\beta} \\ MP_1 &= \frac{\partial Y}{\partial x_1} = \lambda^{\alpha-1+\beta} \alpha A x_1{}^{\alpha-1} x_2{}^{\beta} \qquad (A.9) \\ MP_1 &= \frac{\partial Y}{\partial x_1} = \lambda^{0}[\alpha A x_1{}^{\alpha-1} x_2{}^{\beta}] \end{aligned}$$

Thus, multiplying both inputs by λ leaves the marginal product of x_1 unchanged. In other words the marginal products are zero degree homogeneous in inputs. The same outcome can be proved for input x_2. Since the MRTS is the ratio of MPs, then proportional increases in both inputs by the scalar value λ (implying higher isoquant levels) have no affect on the MRS. Thus, a ray from the origin must cut all isoquants (indifference) curves at points of equal slope. Green (1971) states that the isoquants (indifference) curves are therefore 'homothetic with respect to the origin' (pp141).

As a result of this property, Allanson (1989) notes that,

'optimal factor (commodity) allocations are independent of the level of (aggregate) output (income)' (pp.1).

Increases in the level of aggregate output (utility) with relative input (commodity) price ratios fixed has no affect on factor intensity since the *expansion path* is a straight line from the origin.[20] Moreover, the assumption of weak separability ensures that the introduction of other inputs (commodities) not in the aggregator function also has no consequence for factor (commodity) usage ratios. Hence, changes in input (commodity) intensities x_i will only be a function of the relative prices of various types of input x_i in that part of the nest.

Allanson (1989) also notes that relative price changes in one nest can have *indirect* effects on input (commodity) allocations elsewhere in the nest. Referring to the nested structure in Figure A1, if the price of input x_2 increases, this will affect the optimal combination of x_1 and x_2 in the aggregate nest, but due to the separability restriction, it will not directly affect the optimal use of x_3. There will, however, be an *indirect* effect on the use of x_3 due to a rise in the composite price of *aggregate* input X. This implies that the firm will substitute x_3 for aggregate X in the top nest. Moreover, if x_3 was an aggregate input, then as a consequence of linear homogeneity, its increased use would be translated proportionally to all inputs in that nest.

Thus, if expression (A.1) satisfies both weak separability and linear homogeneity, then the underlying production function is said to be *weakly homothetically separable* (or 'homogenously separable' Green 1971, pp.152-156) and ensures *consistent aggregation.*[21] Consistent aggregation makes it possible to index correctly over prices and quantities when forming composites such that multi-stage nested optimisation procedures give equivalent results to single stage optimisation problems (Ozanne, 1992).

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[20] Increases in aggregate output (utility) are movements onto higher isoquants (indifference) curves; Expansion paths join points of cost minimising equilibria.

[21] It is important to note that weak homothetic separability does not imply that the production function itself is homothetic.

Appendix B: A Linearised representation of a nested production function

Consider a 2-stage nested production function (this approach can also be applied to a utility function), where final output is a Leontief function of a ‘composite’ intermediate input and composite primary factor. In the lower portion of the nest, the composite input/primary factor is subdivided into specific types ‘i’. The intermediate input nest is characterised using CD substitution possibilities, and the value added nest is specified as CES.

The aim of the exercise is to present a range of possible linearised functional forms typically used in nested CGE model structures. Moreover, it will provide some insight into the interpretation of linearised functions which will be employed freely in the discussion in subsequent chapters.

Notation

$Z_k \Rightarrow$ Output in industry ‘k’.

$P_k \Rightarrow$ The output price in industry ‘k’.

$Y_{j,k} \Rightarrow$ Demand for the composite intermediate input, ‘j’ in industry ‘k’.

$W_{j,k} \Rightarrow$ The input price of composite intermediate input, ‘j’ in industry ‘k’.

$X_{j,k} \Rightarrow$ Demand for value added composite, ‘j’ in industry ‘k’.

$U_{j,k} \Rightarrow$ The input price of composite primary factor, ‘j’ in industry ‘k’.

$T_{i,j,k} \Rightarrow$ Input demand for intermediate input of type ‘i’, in composite intermediate input nest ‘j’ in industry ‘k’.

$F_{i,j,k} \Rightarrow$ The price of intermediate input ‘i’.

$V_{i,j,k} \Rightarrow$ Input demand for primary factor of type ‘i’, in composite value added nest ‘j’, in industry ‘k’.

$R_{i,j,k} \Rightarrow$ The price of primary factor ‘i’.

Lower case letters are the percentage change equivalent of the upper case ‘levels’ variable.

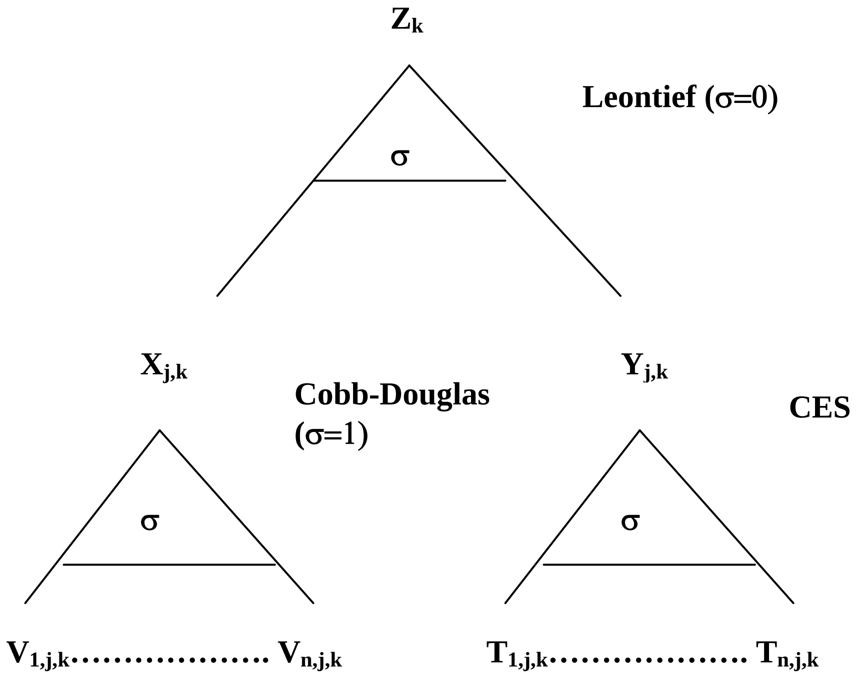

Figure B1: Schematic representation of the production nest.

Mathematical derivations of linearised nested demand functions

Composite Input Nest

This appendix is based on the mathematical techniques provided in Dixon et al., (1992). The top nest in the tree is by definition a single production process Leontief structure. Hence, assuming rationality on the part of producers, levels demands for composite inputs are restricted by a fixed share coefficient. Composite intermediate and primary factor demands are given in equation B1:

$$Y_{j,k} = \gamma_{j,k} Z_k \qquad X_{j,k} = \gamma_{j,k} Z_k \tag{B1}$$

where $\gamma_{j,k}$ are the fixed input-output coefficients. Linearised Leontief demands are given as:

$$y_{j,k} = z_k \qquad x_{j,k} = z_k \tag{B2}$$

Note that the absence of any price effects is due to the zero value of the elasticity of substitution. Hence, increases in output are translated as equiproportional changes in demands for each composite intermediate input which implies CRS. Assuming zero profit:

$$P_k Z_k = W_{j,k} Y_{j,k} + U_{j,k} X_{j,k} \tag{B3}$$

Substituting demands in (B1) into (B3) and simplifying:

$$P_k = \gamma_{j,k} W_{j,k} + \gamma_{j,k} U_{j,k} \tag{B4}$$

Linearising gives a composite price of:

$$p_k = S_{1,k} w_{j,k} + S_{2,k} u_{2,k} \tag{B5}$$

where $S_{j,k}$ is an output share weighted by price, where for the composite intermediate input:

$$S_{1,k} = \frac{\gamma_{1,k} W_{1,k}}{\gamma_{1,k} W_{1,k} + \gamma_{2,k} U_{2,k}} \tag{B6}$$

Primary Factor Nest

In the primary factor nest, production is characterised by a CRS CES function:

$$X_{j,k} = A_{j,k} \left[\sum_{i=1}^{n} \delta_{i,j,k} V_{i,j,k}^{-\rho} \right]^{-\frac{1}{\rho}} \tag{B7}$$

where $A_{j,k}$ is a scale parameter, $\delta_{i,j,k}$ is a distribution share parameter and ρ is an elasticity parameter. Minimising cost subject to (B7) gives first order conditions:

$$R_{i,j,k} = \Lambda A_{j,k} \left[\sum_{i=1}^{n} \delta_{i,j,k} V_{i,j,k}^{-\rho} \right]^{-\frac{1+\rho}{\rho}} \delta_{i,j,k} V_{i,j,k}^{-(1+\rho)} \tag{B8}$$

$$X_{j,k} = A_{j,k}\left[\sum_{i=1}^{n}\delta_{i,j,k}V_{i,j,k}^{-\rho}\right]^{-\frac{1}{\rho}} \tag{B9}$$

Substituting (B9) into (B8) simplifies the latter:

$$R_{i,j,k} = \Lambda A_{j,k}^{-\rho} X_{j,k}^{(1+\rho)} \delta_{i,j,k} V_{i,j,k}{}^{-(1+\rho)} \tag{B10}$$

where (B9) and (B10) are the levels first order conditions. Linearisation of (B9) gives:

$$x_{j,k} = \sum_{i=1}^{n} S_{i,j,k} v_{i,j,k} \tag{B11}$$

where

$$S_{l,j,k} = \frac{\delta_{l,j,k}V_{l,j,k}^{-\rho}}{\sum_{i=1}^{n}\delta_{i,j,k}V_{i,j,k}^{-\rho}} \tag{B12}$$

Substituting (B10) into the input expenditure share formula (B13) in the intermediate nest:

$$\frac{R_{1,j,k}V_{1,j,k}}{\sum_{i=1}^{n} R_{i,j,k}V_{i,j,k}} \tag{B13}$$

and cancelling terms shows the equivalence of expressions (B12) and (B13). This alternative form of the share $S_{i,j,k}$ avoids the process of calibration since it eliminates distribution parameter $\delta_{i,j,k}$ where the shares are merely updated by the percentage changes in prices and quantities. Linearisation of (B10) gives:

$$r_{i,j,k} = \lambda + (1+\rho)x_{j,k} - (1+\rho)v_{i,j,k} \tag{B14}$$

Thus, equations (B11) and (B14) are linearised first order conditions, where r, x, v and λ are percentage changes in R, X, V and Λ.

Rearrange (B14) in terms of $v_{i,j,k}$ gives:

$$v_{i,j,k} = -\sigma r_{i,j,k} + \sigma\lambda + x_{j,k} \qquad \text{(B15)}$$

where σ is the elasticity of substitution between all pairwise types of primary factors (i.e. labour, capital) in the value added nest:

$$\sigma = \frac{1}{1+\rho} \qquad \text{(B16)}$$

substituting (B15) into (B11) and rearranging in terms of σλ yields:

$$\sigma\lambda = \sigma\sum_{i=1}^{n} S_{i,j,k} r_{i,j,k} \qquad \text{(B17)}$$

Substituting (B17) into (B15) eliminates the percentage change Lagrangian variable λ. Factorising the resulting expression gives linearised CES Hicksian primary factor demands:

$$v_{i,j,k} = x_{j,k} - \sigma\left[r_{i,j,k} - \sum_{i=1}^{n} S_{i,j,k} r_{i,j,k} \right] \qquad \text{(B18)}$$

For consistent aggregation expression (B19) must hold:

$$U_{j,k} X_{j,k} = \sum_{i=1}^{n} R_{i,j,k} V_{i,j,k} \qquad \text{(B19)}$$

By linearising (B19), substituting (B11) and rearranging, it is possible to derive the percentage change in the composite price in the value added nest as:

$$u_{j,k} = \sum_{i=1}^{n} S_{i,j,k} r_{i,j,k} \qquad \text{(B20)}$$

Further substitution of (B20) into (B18) gives a simplified version of the linearised Hicksian demand function:

$$v_{i,j,k} = x_{j,k} - \sigma\left[r_{i,j,k} - u_{j,k}\right] \qquad \text{(B21)}$$

Hence, equation (B21) shows how the demand for primary input 'i' can be broken into an expansion (or output) effect ($x_{j,k}$) and a price effect, the size of which is governed by the extraneous elasticity of substitution parameter, σ. The proportionality of changes in aggregated primary factor usage on each type 'i' is a reflection of constant returns to scale in the aggregator function. Moreover, any increase in the price of factor 'i' ($r_{i,j,k}$), relative to the composite price index ($u_{j,k}$), leads to reduced usage of primary factor 'i' relative to other primary factors in the nest. The size of this price substitution effect is dependent on the magnitude of the elasticity of substitution.

Intermediate Input Nest

The choice of functional form for the characterisation of intermediate input demands is a generalised Cobb-Douglas:

$$Y_{j,k} = B_{j,k}\prod_{i=1}^{n} T_{i,j,k}^{\alpha_{i,j,k}} \qquad \text{(B22)}$$

where minimisation of cost subject to the production function (B22) gives the Lagrangian:

$$Z = \sum_{i=1}^{n} F_{i,j,k}T_{i,j,k} + \Lambda(Y_{j,k} - B_{j,k}\prod_{i=1}^{n} T_{i,j,k}^{\alpha_{i,j,k}}) \qquad \text{(B23)}$$

Using the same principles as above gives first order linearised conditions:

$$f_{n,j,k} = \lambda + y_{j,k} - t_{n,j,k} \qquad \text{(B24)}$$

$$y_{j,k} = \sum_{i=1}^{n} \alpha_{i,j,k} t_{i,j,k} \qquad \text{(B25)}$$

where the α parameters are cost shares (summing to one). Using the same methodology to solve first order linearised conditions gives Hicksian linearised Cobb-Douglas intermediate input demands:

$$t_{n,j,k} = y_{j,k} - \left[f_{n,j,k} - \sum_{i=1}^{n} \alpha_{i,j,k} f_{i,j,k} \right] \qquad \text{(B26)}$$

Given consistent aggregation in the nest, the following accounting identity must hold:

$$W_{i,j,k} Y_{i,j,k} = \sum_{i=1}^{n} F_{i,j,k} T_{i,.j,k} \qquad \text{(B27)}$$

Linearising (B27), substituting (B25) and rearranging in terms of $w_{j,k}$ gives the linearised composite intermediate input price in the nest:

$$w_{j,k} = \sum_{i=1}^{n} \alpha_{i,j,k} f_{i,j,k} \qquad \text{(B28)}$$

Substituting (B28) into (B26) gives a simplified version of the Cobb-Douglas Hicksian demands for intermediate input 'i':

$$t_{n,j,k} = y_{j,k} - \lfloor f_{n,j,k} - w_{j,k} \rfloor \qquad \text{(B29)}$$

This linearised demand function has exactly the same interpretation as the CES primary factor demands. The unitary value of the elasticity of substitution parameter is implicitly recognised within the price effect component of the demand function.

Summary of Production Nest Input Demands

Composite Input/Factor Demands (Leontief)

$$y_{j,k} = z_k \qquad \text{(B2)}$$

$$x_{j,k} = z_k \qquad \text{(B2)}$$

Composite price in the nest:

$$p_k = S_{1,k} w_{j,k} + S_{2,k} u_{2,k} \qquad \text{(B5)}$$

Primary Factor Demands (CES):

$$v_{i,j,k} = x_{j,k} - \sigma \left\lfloor r_{i,j,k} - u_{j,k} \right\rfloor \tag{B21}$$

Composite price in the nest

$$u_{j,k} = \sum_{i=1}^{n} S_{i,j,k} r_{i,j,k} \tag{B20}$$

Intermediate Input Demands (Cobb-Douglas):

$$t_{n,j,k} = y_{j,k} - \left\lfloor f_{n,j,k} - w_{j,k} \right\rfloor \tag{B29}$$

Composite price in the nest:

$$w_{j,k} = \sum_{i=1}^{n} \alpha_{i,j,k} f_{i,j,k} \tag{B28}$$