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The impacts of nonlinearities and shifts in ecosystems in optimal groundwater pumping patterns

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Abstract
Groundwater management is currently one of the main issues in environmental regulation worldwide. The large depletion and contamination of these resources during recent decades, calls for the implementation of control measures capable of reduce or dampen down the escalating global depletion, which is above 150 Mm^3 per year. While several efforts have been done analyzing suitable regulations and measures to control extractions and protect these resources, the impacts of groundwater resources over their dependent aquatic ecosystems is still a pending issue. The large degradation of ecosystems, especially fresh water biota and habitats, requires the protection of specific ecosystems and their habitats. During recent decades, the interest on studying ecosystems has increased exponentially, with much more accurate analysis of the behavioral response of ecosystems to changes in their habitat. A broad literature suggests that the existence of nonlinearities in ecosystem behavior seems to be most precise representation of ecosystem status. Additionally, the existence of shifts in ecosystem's behavior due to large deterioration of their external conditions appears to be a quite frequent feature. This paper purpose is to include ecosystems as very important elements to be considered in groundwater management. The contribution of this paper is the inclusion of nonlinearities and switches in the ecosystem' status due to habitat deterioration (groundwater depletion). Using a two-stage optimal control method, we obtain the optimal patterns of irrigation water extractions and water table levels. The results highlight how both the economic value and the ecosystem' status functions have important impacts on water extractions and groundwater levels. Additionally, the existence of shifts in ecosystem's behavior affects critically the characterization of the optimal groundwater strategy.

Keywords: freshwater ecosystems; ecosystems behavior function; nonlinear ecosystems; shifts in ecosystems; groundwater management; two-stage optimal control model.

JEL Codes: Q25, Q57, Q58.

1. Introduction

Since the second half of the XX century, the exponential growth of population and income, and the resulting change in consumption's patterns have triggered massive pressures over the environment. Human activities have become the major driver of worldwide natural resources deterioration. Problems such as water contamination and depletion, air quality deterioration, erosion and loss of soils, or biodiversity and ecosystems losses are some of the most important environmental problems worldwide. Additionally, global warming will bring substantial changes in weather patterns (temperatures and precipitations) with more frequent extreme events and further degradation of the environment and the natural resources.

Ecosystems consist of living organisms and their interactions that are adapted to specific habitats. Ecosystems provide a large set of good and services which are essential to the maintenance of human activities and societies. However, the large pressures over natural resources, with substantial alterations in ecosystems' habitats and increasing degradation and exploitation of ecosystems, is threatening their survival. The Global Living Planet Index (WWF, 2014) shows a decline of 52% in vertebrate species population since the 1970s. Freshwater ecosystems are the most impacted systems with a decline in population of around 76% (WWF, 2014), and 65% of the aquatic habitats supported by river flows being classified as moderately to highly threatened (Davis et al., 2015: Vörösmarty et al., 2010). The main threats to freshwater ecosystems are associated with human activities ending up in ecosystems' habitat degradation and depletion because of reduced streamflows and large pollution loads degrading freshwater resources (Collen et al., 2014). Additionally, several studies have also reported large impacts on ecosystems and biodiversity driven by climate change (Walter et al., 2002; Leemans and Eickhout 2010; Woodward et al., 2010). The protection of freshwater ecosystems requires, not just a better knowledge of the interactions and behavior of these ecosystems, but also the defense and restoration of freshwater bodies.

Freshwater is considered one of the most important resources on the Earth, essential to maintain human and ecosystems activities and their survival (Vörösmarty et al. 2010). Most freshwater resources on the planet are collected or linked to aquifer systems. Underground water systems are the main source of human drinking water and the major support of agricultural activities in several regions of the world. Additionally, these resources are the main feeder of habitats supporting freshwater ecosystems. However, over the last 50 years these resources are being put under unprecedented pressure (Konikow, 2011; Wada et al., 2010). Despite the large importance of these resources, most of the aquifers worldwide are currently mismanaged. The application of environmental regulations to control and manage these resources are facing several problems of implementation and control, which hinder the correct protection of groundwater resources. The scarcity of information on groundwater status and the complexity of groundwater systems hamper the efficient implementation and operation of regulations (van Engelenburg et al., 2018; Figureau et al., 2015).

Due to the great importance of groundwater resources, a large set of literature deals with the analysis of efficient paths to manage these resources (see the reviews by Koundouri (2004) and Koundouri et al. (2017)). However, the effective management and control of groundwater resources is still far from being achieved. Furthermore, many studies on groundwater management ignore the relationships between these resources and dependent ecosystems, which can not survive when aquifers are depleted. Recent studies analyze or categorize groundwater dependent ecosystems and their connections and interactions with groundwater resources (e.g., Tuinstra and van Wensem, 2014; Klove et al., 2011a; Klove et al., 2011b; Howard and Merrifield, 2010; Hancock et al. 2008; Eamus and Froend, 2006). By contrast, other studies focus on how quality and quantity problems are affecting ecosystem's habitats and its survival (Goedhart and Pataki, 2011; Whiteman et al., 2010; Elmore et al., 2003). Finally, a string of the literature includes ecosystems functions and services in groundwater management (Pongkijvorasin et al., 2018; Gutrich et al., 2016; Esteban and Dinar, 2016; Esteban and Albiac, 2011).

The purpose of this paper is to extend groundwater management literature by analyzing how shifts and nonlinearities in ecosystems' behavior alter the optimal groundwater extractions paths. Previous literature has already demonstrated that the inclusion of ecosystems, as an additional groundwater user, changes the optimal groundwater strategy (Esteban and Dinar, 2016; Esteban and Albiac, 2011). However, these analyses are based on linear ecosystem behavior functions where the status of the ecosystem does not change rapidly even though the habitat is largely deteriorated. In this article we extend this analysis by including nonlinearities and shifts in the ecosystems' status which seems to be a more realistic assumption on the biological behavior of ecosystems (e.g., Beisner et al., 2003; Suding et al., 2004; Scheffer et al., 2001).

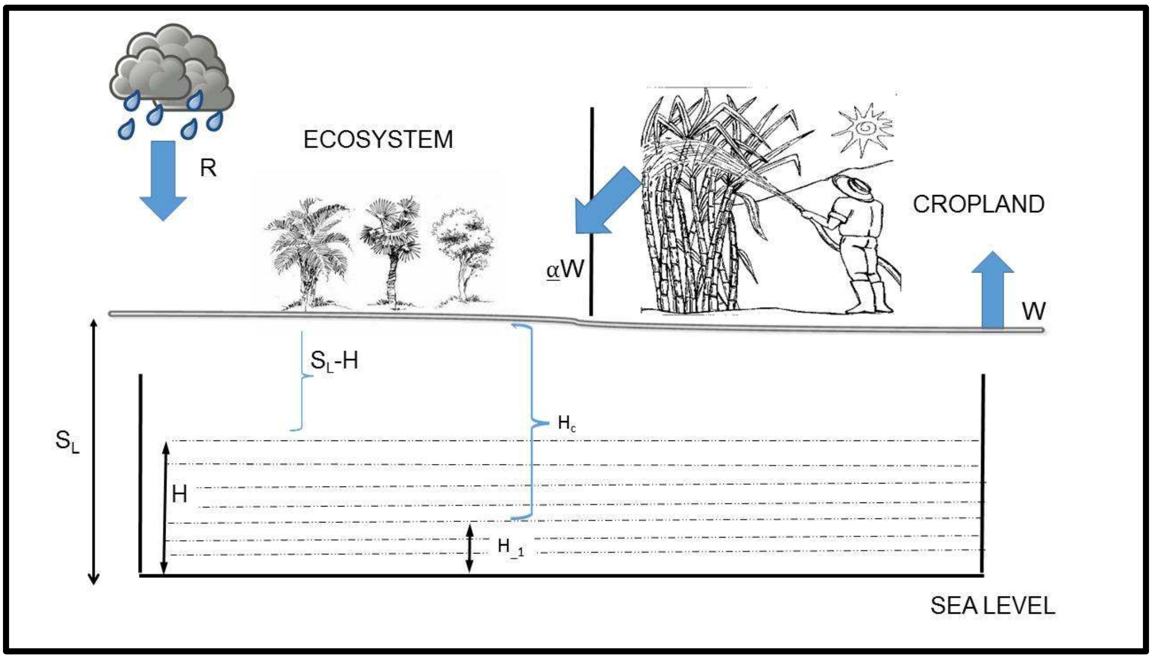
As expected, the results show how the inclusion of nonlinear ecosystems' behavior and predictable changes in their status are very relevant elements in groundwater management. This analysis contributes to a better understanding of the relationship between groundwater and ecosystems, and can be quite helpful for the design and implementation of groundwater policies. A sustainable use of groundwater resources requires coordinated actions to protect these resources, taking into account the impacts and links between groundwater and ecosystems.

Following, in section 2 we present a traditional groundwater model and we establish a nonlinear ecosystem' function with a critical threshold once a specific water table level is exceeded. In section 3 we solve the mathematical optimal control problem by implementing the 'two-stage' optimal control method. Finally, section 4 concludes.

2. Groundwater model with nonlinearities and shifts in ecosystems: model equations

We state a typical groundwater model based on previous literature (e.g., Gisser and Sanchez 1980, 1982; Fienerman and Knapp, 1983; Brill and Burness, 1994). Traditionally, groundwater models assume a single-cell aquifer with a flat bottom and a fix natural recharge (R). The aquifer resources are being exclusively used for irrigation purposes, where equal irrigators extract water to maintain their economic activities (W_t). The total water in the aquifer is represented by the difference between the surface level (S_L) and the aquifer water table level (H_t). A simple illustration of an aquifer is represented in Figure 1.

Figure 1. Traditional aquifer representation



The groundwater budget ($\dot{H}$) is a function of the natural recharge (R) minus net water extractions $(1-\propto)\cdot W_t$. Where net water extractions are the total withdrawals (W_t) minus the rate of water that infiltrates again to the aquifer after irrigation use ($\propto$).

(1) $\dot{H} = \frac{1}{AS}\cdot(R-(\propto -1)\cdot W_t)$

The total groundwater available (AS) is a ratio between the aquifer's volume multiplied by the storativity level, which represents the unconfined part of the aquifer or the water effectively available.

This traditional model has been taken as reference, but including the existence a groundwater user, irrigators, that extracts water to maintain their private economic income and the existence of some aquatic ecosystems which survival is conditioned by the aquifer water level. The social planner problem consists in maximizing the net present value of future income streams from both the private irrigators ($I(W_t, H_t)$) and the environment ($E(H_t)$). The model can be stated as following:

(2) $$\underset{W_t, H_t}{\text{Max}} \int_0^{\infty} exp^{-rt}[I(W_t, H_t) + E(H_t)]dt$$

with W_t and H_t being, respectively, total groundwater extractions (control variable) and the groundwater table level (state variable). r is the discount rate, which is a positive parameter. This maximization is constrained to the dynamic of the resource (eq. 1), the initial condition of the water table level $H(0) = H_0$, and the positive value of the water table level $H_t \geq 0$.

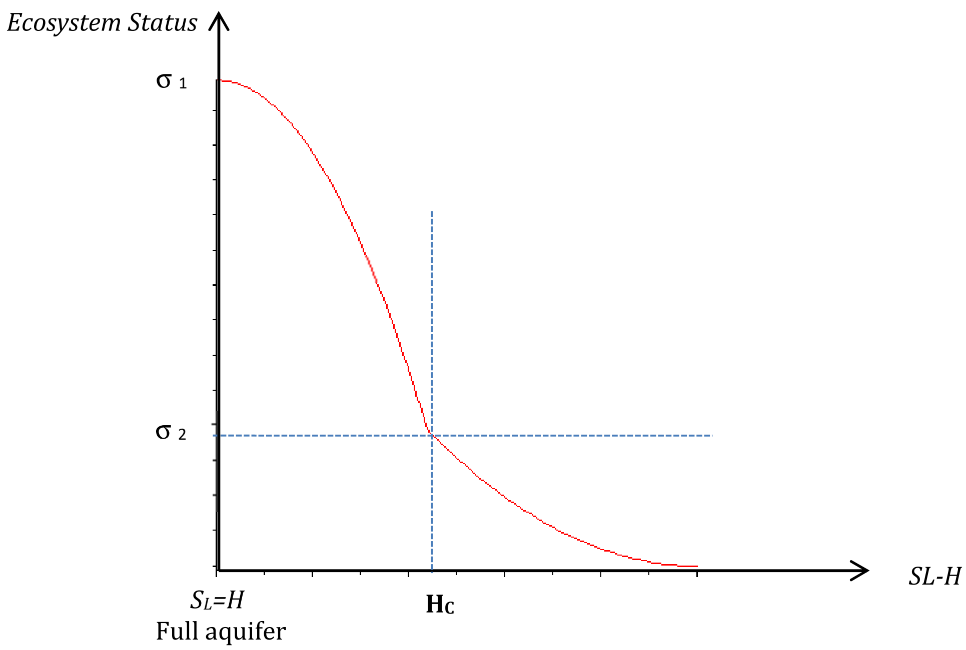
The irrigators' private income follows the same specification of previous works based on a quadratic benefit function minus the pumping costs that depends on fix costs and the marginal pumping costs (Koundouri et al., 2017).

(3) $$I(W_t, H_t) = B(W_t) - C(W_t, H_t) = \frac{1}{2k} \cdot W^2 - \frac{g}{k} \cdot W - (C_0 - C_1 H)W$$

where private benefits from groundwater extractions ($B(W_t)$) come from a linear specification of a groundwater demand function $W = g + k \cdot P$ (being P the groundwater price, g and k parameters, with $g > 0$ and $k < 0$). The total costs of groundwater extractions ($C(W_t, H_t)$) depend on the total withdrawals and the distance of the water table level, with c_0 being a fix costs and c_1 being the marginal costs from groundwater extractions.

On the other hand, this model includes the existence of several aquatic ecosystems which survival is linked with the groundwater resources, $E(H_t)$. We assume that the behavior of this general ecosystem, or habitat, is represented by a quadratic nonlinear function. This function represents the fact that the lower the water table level (H_t), the more difficult the survival of the ecosystem[1] (see Fig. 2), with the ecosystem health decreasing at a non-constant rate. Additionally, as has been largely supported by the literature, the model includes a critical threshold that switch the behavior of the ecosystem once it is exceeded (H_C). We also assume that whereas the ecosystem can largely reduce its 'health' or status, it will not totally disappear. This type of ecosystem's formulation has been largely stated in the literature as a typical behavior of several ecosystems and habitats (Scheffer et al., 2001; Hughes et al., 2013). Despite there are controversy in the literature on the empirical evidence of nonlinearities on freshwater ecosystems' behavior (Capon et al., 2015), nonlinearities and shifts in freshwater ecosystems have been supported in the literature (Heffernan, 2008; Utz et al., 2008).

Figure 2. Nonlinear ecosystem function



Note: H_C is the ecosystem critical threshold. Once this level of the water table level is reached the ecosystem state switches to a different status. When the aquifer is full ($S_L = H$) the status or health of the ecosystem is at maximum.

[1] We assume a general function that can represent several issues as habitat biodiversity, total number of species, number of members into a specific population, etc.

Following the previous ecosystem representation stated on Figure 2 the mathematical expression can be stated as follows:[2]

$$(4) \qquad EC(H_t) = \begin{cases} \frac{(\sigma_2-\sigma_1)\cdot(S_L-H_t)^2}{H_c^2} + \sigma_1 & if\ (S_L - H_t) \leq H_c \\ A \cdot (S_L - H_t)^2 + B \cdot (S_L - H_t) + C & if\ (S_L - H_t) > H_c \end{cases}$$

where σ_1 and σ_2 are the positive parable parameters, $(S_L - H_t)$ represents the aquifer depletion, and $A = \frac{\sigma_2}{(S_L-H_c)^2}$, $B = -\frac{2\, S_L\cdot\sigma_2}{(S_L-H_c)^2}$, $C = \frac{S_L^2\cdot\sigma_2}{(S_L-H_c)^2}$. The parable parameters have been defined by assuming that in the point where the critical threshold (H_C) is reached the ecosystem status has the same value both right and left the function. Additionally, we assume that when the aquifer is totally depleted $(H_t = 0)$ the ecosystem is extinguished. The total economic value of the ecosystem is:

$$(5) \qquad E(H_t) = \varphi \cdot EC(H_t)$$

We establish that this ecosystem provides several goods and services to society that contributes with the social welfare. We state that the aggregate economic value of the ecosystem is a constant (φ). The greater the total welfare or health of the ecosystem the higher the benefits to society.

3. Optimal groundwater management patterns under nonlinearities and switches in the ecosystem status

Assuming the equations established in previous section 2 the resolution of the social planner problem requires the implementation of more sophisticated optimization tools. Ordinary optimal control theory states the resolution of

[2] We have included a parable as a functional equation of the ecosystem behavior in order to obtain mathematical solutions in the optimal control problem.

dynamic problems by a single optimization phase (Amit, 1986). However, when abrupt changes occur and the problem's objective function suddenly switches to a different status, which prevail during a certain time, the problem requires different phases to get an optimal solution ('multi-stage' dynamic optimization problems). '*An optimal multiprocess problem is a dynamic optimization problem involving a collection of control systems coupled through constraints in the endpoint of the state trajectories…'* (Babad, 1995).

In this case, the social planner maximization becomes in a 'two-stage' optimal control problem that need to be solved recursively. We denoted the infinite horizon discounted two-stage optimal control problem as *SW*, and we can rewrite problem (2) as follows:

$$(6) \qquad Max \;\; SW(W_t, H_t, t_1) =$$

$$\int_{t_0}^{t_1} e^{-rt} \cdot \left[I(W_t, H_t) + \xi \cdot \left(\frac{(\sigma_2 - \sigma_1)(S_L - H_t)^2}{H_c^2} + \sigma_1 \right) \right] dt +$$

$$\int_{t_1}^{\infty} e^{-rt} \cdot [I(W_t, H_t) + \xi \cdot (A \cdot (S_L - H_t)^2 + B \cdot (S_L - H_t) + C)] dt$$

$$s.t. \qquad \dot{H} = \frac{[R + (\alpha - 1)W]}{AS}$$

$$H(t) \geq 0 \quad free$$

$$H(0) = H_0 \quad and \quad H(1) = H_{\_1} \quad given$$

$$t_1 \quad free$$

A, *B*, and *C* are the ecosystem function's parameters (eq. 4), and $H_{\_1}$ being the critical water table level at which the threshold is reached ($(S_L - H_{\_1}) = H_c$).

This problem represents existence of different objective function depending on the status of the ecosystem, which switches when a threshold (H_C) is reached at a time t_1. The resolution of the 'two-stage' optimal control problem, corresponding with the two ecosystem regimes, is solved by imposing a sequence of two phases with two different Pontryagin problems (Tomiyama, 1985; Tomiyama and Rossana, 1989; Makri, 2004; Aisa et al., 2007). The 'two-stage' method proceeds by backward solving the two problems.

The optimal control problem (SP_2) is primarily solved by:

(7) $$Max\ \ SP_2(W_2, H_2, t_1) = \int_{t_1}^{\infty} e^{-rt} \cdot [I(W_2, H_2) + \xi \cdot (\, A \cdot (S_L - H_2)^2 + B \cdot (S_L - H_2) + C)]dt$$

$$s.t. \qquad \dot{H}_2 = \frac{[R + (\alpha - 1) \cdot W_2]}{AS}$$

$$H_2(t_1) = H_{\_1} \quad given$$

$$t_1 \quad free$$

with W_2 and H_2 being respectively the optimal solutions of the water extractions and the water table level under the Pontryagin problem corresponding to the sub-problem 2 (SP_2).

The Hamiltonian associated to the first Pontryagin problem is:

(8) $$\mathcal{H}_2(t, W_2, H_2, \lambda_2) = -e^{-rt}[I(W_2, H_2) + \xi \cdot (A(S_L - H_2)^2 + B(S_L - H_2) + C)] + \lambda_2 \cdot \frac{[R+(\alpha-1)\cdot W_2]}{AS}$$

with λ_2 being the co-estate variable of the sub-problem 2. By solving the first order conditions we obtain the optimal results of this problem ($SP_2^*(W_2^*, H_2^*, t_1)$), and the optimal values of the state and control variables (W_2^* and H_2^* respectively):[3]

(9) $$W_2^*(t) = \bar{B} \cdot e^{t \cdot x_2} - \frac{M}{n}$$

(10) $$H_2^*(t) = \frac{m \cdot \bar{B}}{x_2} \cdot e^{t \cdot x_2} + \frac{\left(r \cdot \frac{M}{m} - N\right)}{n}$$

[3] A complete resolution of the first part (phase 1) of the two-state optimal control problem is placed on the Appendix.

with where $m = \frac{(\alpha-1)}{AS}$, $M = \frac{R}{AS}$, $n = -r \cdot C_1 \cdot k + 2 \cdot \xi \cdot \mathrm{A} \cdot m \cdot k$, and $N = \big(-r \cdot g - C_0 \cdot k \cdot r + k \cdot C_1 \cdot M - m \cdot k \cdot \xi \cdot (2 \cdot A \cdot S_L + B)\big)$. Additionally, $x_2 = \frac{r-(r^2+4 \cdot n \cdot m)^{1/2}}{2}$ is the negative root of the polynomial equation from the system of two differential equations established to obtain the optimal solutions of the problem (see Appendix, eq. *A11*). Finally, the coefficient $\bar{B}$ is calculated using the initial conditions of the problem, and can be expressed as:

$$(11) \qquad \bar{B} = \frac{x_2}{m} . e^{-t_1 \cdot x_2} . \left(H_{\_1} - \frac{\left(r \cdot \frac{M}{m} - N\right)}{n} \right)$$

Having the optimal value of the SP_2, the second phase is the optimization of the second Pontryagin problem that correspond with the first sub-problem (SP_1):

$$(12) \qquad Max \;\; SP_1(W_1, H_1, t_1) = \int_0^{t_1} e^{-rt} \cdot \left[I(W_2, H_2) + \xi \cdot \left(\frac{(\sigma_2 - \sigma_1) \cdot (S_L - H_1)^2}{H_C^2} + \sigma_1 \right) \right] dt + SP_2^*(W_2^*(t_1), H_2^*(t_1), t_1)$$

$$s.t. \qquad \dot{H}_1 = \frac{[R + (\alpha - 1) \cdot W_1]}{AS}$$

$$H_1(0) = H_0 \quad given$$

$$H_1(t_1) = H_{\_1} \quad given$$

$$t_1 \quad free$$

with W_1 and H_1 being the solutions, water pumping and water table level respectively, under the sub-problem 1 (SP_1).

The Hamiltonian associated with the second Pontryagin problem is:

$$(13) \; \mathcal{H}_1(t, W_1, H_1, \lambda_1) = -e^{-rt} \left[I(W_1, H_1) + \xi \cdot \left(\frac{(\sigma_2-\sigma_1)(S_L-H_1)^2}{H_C^2} + \sigma_1 \right) \right] + \lambda_1 \cdot \frac{[R+(\alpha-1) \cdot W_1]}{AS}$$

with λ_1 being the co-estate variable of the sub-problem 1.

By solving the first order conditions and imposing the continuity condition where $\lambda_1^*(W_1^*(t_1), H_1^*(t_1), t_1^*) = \lambda_2^*(W_2^*(t_1), H_2^*(t_1), t_1^*)$ and the transversality condition $\mathcal{H}_1(t_1) = \frac{\partial SP_2^*(W_2^*(t_1), H_2^*(t_1), t_1)}{\partial t_1}$ we obtain the optimal solutions for the pumping extractions, the water table level, and the optimal time (t_1^*). For a detailed resolution of the problem see Appendix.

$$(14) \qquad W_1^*(t) = \overline{CA} \cdot e^{ty_1} + \overline{CB} \cdot e^{ty_2} \quad - \frac{M}{m}$$

$$(15) \qquad H_1^*(t) = \frac{m \cdot \overline{CA}}{y_1} \cdot e^{ty_1} + \frac{m \cdot \overline{CB}}{y_2} \cdot e^{ty_2} + \frac{(r \cdot \frac{M}{m} - NN}{nn})$$

the parameters $\overline{CA}$ and $\overline{CB}$ are the equations´ constants determined with restrictions $H_1(0) = H_0$ and $H_1(t_1) = H_{\_1}$ of the sub-problem 1 (*SP1*). Additionally, $m = \frac{(\alpha - 1)}{AS}$, $M = \frac{R}{AS}$, $nn = -C_1 \cdot k \cdot r + 2 \cdot \xi \cdot k \cdot m \cdot \frac{(\sigma_2 - \sigma_1)}{H_c^2}$, and $NN = -r \cdot g - C_0 \cdot k \cdot r + k \cdot C_1 \cdot M - 2 \cdot \xi \cdot k \cdot m \cdot \frac{(\sigma_2 - \sigma_1)}{H_c^2} \cdot S_L$. Finally, y_1 and y_2 are the roots of the polynomial equation $(y^2 - r \cdot y - nn \cdot m = 0)$ established to solve the problem[4]. The value of the constants $\overline{CA}$ and $\overline{CB}$ can be expressed as:

$$(16) \qquad \overline{CA} = \frac{H_{\_1} - H_0 e^{t_1 y_2} + \left(\frac{r \cdot \frac{M}{m} - NN}{nn}\right) e^{t_1 y_2}}{\frac{m \cdot}{y_1} \cdot (e^{t_1 y_1} - \cdot e^{t_1 y_2})}$$

$$(17) \qquad \overline{CB} = \frac{y_2}{m} \cdot \left(H_0 - \left(\frac{r \cdot \frac{M}{m} - NN}{nn}\right) - \frac{H_1 - H_0 e^{t_1 y_2} + \left(\frac{r \cdot \frac{M}{m} - NN}{nn}\right) e^{t_1 y_2}}{(e^{t_1 y_1} - e^{t_1 y_2})} \right)$$

The optimal paths of the optimal water extraction level (W^*) and the optimal water table level depend on the solutions of the two sub-problems stated on eqs. 7 and 12. The main conclusions we can extract from the expressions is that both variables largely depend on the economic value of the ecosystem (ξ), the

[4] See Appendix for a detailed explanation.

specification of the ecosystem's function ($EC(H_t)$), and finally the threshold where the ecosystem shifts from one state other (H_C). These results clearly highlight how the determination of the optimal levels of water pumping and water table can be very difficult to establish if a combined policy, that aims to maximizes the social benefits of private income from groundwater pumping and to protect 'sensitive' ecosystems, is implemented.

4. Conclusions

Ecosystems functioning has been mostly disregarded in groundwater management. The large decay in ecosystems during recent decades, especially freshwater habitats, has increased the awareness for protecting these systems. One main factor threatening freshwater ecosystems is the strong deterioration of the quantity and quality of water bodies. In particular, the large overexploitation and depletion of groundwater resources worldwide is one of the major pressures to ecosystems' health.

The study of groundwater resources has been addressed in the literature with growing complexity during the last fifty years. From an economic perspective, the literature since the 1970s has been based on how to achieve an efficient management of these resources to both support economic activities and preserve groundwater bodies. However, the mismanagement of groundwater resources is still a pending issue worldwide, with many aquifer systems under escalating overdraft levels and serious depletion in arid and semi-arid regions. The lack of information on groundwater resources and their physical interactions, the insufficient information on withdrawals and recharges, and the lack of workable control instruments on extractions are elements that hinder the sustainable management of these resources. Additionally, most current regulations to protect these resources do not take into account the existence of dependent ecosystems whose survival is linked to the status of groundwater resources.

In this paper we develop an economic groundwater management model where aquatic ecosystems, which depend on the groundwater status, are included in the optimization problem. The contribution of this paper is the consideration of nonlinear ecosystem responses with rapid changes in ecological status or behavior when critical water level thresholds are reached. In general, linear ecosystems' behavior has been considered a good approximation to explain the performance of ecosystems to habitat changes. However, recent analytical findings show that nonlinear specifications and process regime shifts, including hysteresis, are much better approximations to the behavior of ecosystems (Scheffer et al., 2001). Based on these findings, we model the changes in optimal patterns of water extractions and water table levels when nonlinearities and ecosystems' shifts are taken into account.

The results highlight how the optimal groundwater management is largely conditioned by the ecosystem functions' specification, the economic value of the ecosystem, and the threshold that modify the status of the ecosystem. When ecosystems are not considered, water pumping is frequently very intense leading to substantial depletion and falling water table levels. However, when the nonlinear ecosystem response is considered, the extractions should be curtailed allowing for higher levels of the water table, in order to preserve the ecosystem. These results can be very useful for policymakers in order to implement better policies to control groundwater extractions and maintaining a healthy level for linked ecosystems. Despite the fact that these results can be very useful for implementing enhanced groundwater regulations, better knowledge on ecosystems and their interactions with groundwater bodies are still needed. Future developments call for the empirical illustration of this model results, applied to well-studied aquifers sustaining dependent ecosystems.

5. References

Aisa, R., Cabeza, J., and Larramona, G. 2007. Timing of migration. Economics Bulletin 6, 1-10.

Amit, R. 1986. Petroleum reservoir exploitation: Switching from primary to secondary recovery. Operations Research 34, 534-549.

Babad, H.R. 1995. An infinite-horizon multistage dynamic optimization problem. Journal of optimization theory and applications 86, 529-552.

Beisner, B.E., Haydon, D.T., and Cuddington, K. 2003. Alternative stable states in ecology. Front Ecol Environ 1, 376-382.

Boucekkine, R., Saglam, C., and Vallée, T. 2004. Technology adoption under embodiment: A two-stage optimal control approach. Macroeconomic Dynamics 8, 250-271.

Brill, T.C., and Burness, H.S. 1994. Planning versus competitive rates of groundwater pumping. Water Resource Research 30, 1873-1880.

Capon, S.J., Lynch, A.J.J., Bond, N., Chessman, B.C., Davis, J., Davidson, N., Finlayson, M., Gell, P.A., Hohnberg, D., Humphrey, C., Kingsford, R.T., Nielsen, D., Thomson, J.R., Ward, K., and Mac Nally, R. 2015. Regime shifts, thresholds and multiple stable states in freshwater ecosystems; a critical appraisal of the evidence. Science of the Total Environment 534, 122-130.

Collen, B, Whitton, F., Dyer, E.E., Baillie, J.E.M., Cumerlidge, N., Darwall, W.R.T., Pollock, C., Richman, N.I., Soulsby, A.M., and Böhm, M. 2014. Globa. Patterns of freshwater species diversity, threat and endemism. Global Ecol. Biogeogr. 23, 40-51.

Davis, J., O'Grady, A.P., Dale, A., Arhington, A.H., Gell, P.A., Driver, P.D., Bond, N., Casanova, M., Finlayson, M., Watts, R.J., Capon, S.J., Nagelkerken, I., Tingley, R., Fry, B., Page, T.J., and Specht, A. 2015. When trends intersect: The challenge of protecting freshwater ecosystems under multiple land use and

hydrological intensification scenarios. Science of the Total Environment 534, 65-78.
Eamus, D., and Froend, R. 2006. Groundwater-dependent ecosystems: the where, what and why of GDEs. Australian Journal of Botany 54, 91-96.
Elmore, A.J., Mustard, J.F., and Manning, S.J. 2003. Regional patterns of plant community response to changes in water: Owens Valley, California. Ecological Applications 13, 443-460.
van Engelenburg, J., Hueting, R., Rijpkema, S., Teuling, A.J., Uijlenhoet, R., and Ludwing, F. 2018. Impact of changes in groundwater extractions and climate change on groundwater dependent-ecosystems in a complex hydrogeological setting. Water Resour Manage 32, 259-272.
Esteban E. and Albiac J. 2011. Groundwater and ecosystems damages: questioning the Gisser-Sanchez effect. Ecological Economics 70, 2062-2069.
Esteban, E., and Dinar, A. 2016. The role of groundwater dependent ecosystems in groundwater management. Natural Resource Modeling 29, 98-129.
Feinerman, E., and Knapp, K.C. 1983. Benefits from groundwater management: magnitude, sensitivity, and distribution. American Journal of Agricultural Economics 65, 703-710.
Figureau, A.G., Montginoul, M., and Rinaudo, J.D. 2015. Policy instruments for decentralized management of agricultural groundwater abstraction: A participatory evaluation. Ecological Economics 119, 147-157.
Gisser, M., and Sanchez, D.A. 1980. Competition versus optimal control in groundwater pumping. Water Resources Research 16, 638-642.
Goedhart, C.M., and Pataki, D.E. 2011. Ecosystem effects of groundwater depth in Owens Valley, California. Ecohydrology 4, 458-468.
Gutrich, J.J., Gigliello, K., Gardner, K.V., and Elmore, A.J. 2016. Economic returns of groundwater management sustaining an ecosystem service of dust suppression by alkali meadow in Owens Valley, California. Ecological Economics 121, 1-11.
Hancock, P.J., Hunt, R.J., Boulton, A.J. 2008. Preface: hydrogeoloecology, the interdisciplinary study of groundwater dependent ecosystems. Hydrogeology Journal 17, 1-3.
Heffernan, J.B. 2008. Wetlands as an alternative stable state in desert stream. Ecology 89, 1261-1271.
Howard, J., and Merrifield, M. 2010. Mapping Groundwater Dependent Ecosystems in California. PLOS One 5, e11249.
Hughes, T.P., Linares, C., Dakos, V., van de Leemput, I.A., and van Nes, E.H. 2013. Living dangerously on borrowed time during slow, unrecognized regime shifts. Trends in Ecology & Evolution 26, 149-155.
Leemans, R., and Eickhout, B. 2004. Another reason for concern: regional and global impacts on ecosystems for different levels of climate change. Global Enviornmental Change 14, 219-228.
Makris, M. 2001. Necessary conditions for infinite-horizon discounted two-stage optimal control problems. Journal of Economic Dynamics and Control 25, 1935-1950.
Klove, B., Allan, A., Bertrand, G., Druzynska, E., Erturk, A., Goldscheider, N., Henry, S., Karakaya, N., Karjalainen, T.P., Koundouri, P., Kupfersberger, H., Kvoerner, J.,

Lundberg, A., Muotka, T., Preda, E., Pulido-Velazquez, M., and Schipper, P. 2011. Groundwater dependent ecosystems. Part II. Ecosystem services and management in Europe under risk of climate change and land use intensification. Environmental Science & Policy, 14, 782-793.

Koundouri, P. 2004. Current issues in the economics of groundwater resource management. Water Resour Res 16, 703-740.

Koundouiry, P., Roseta-Palma, C., and Englezo, N. 2017. Out of sight, not out of mind: developments in economic models of groundwater management. International review of environmental and resource economics 11, 55-96.

Pongkijvorasin, S., Burnett, K., and Wada, C. 2018. Joint management of an interconnected coastal aquifer and invasive tree. Ecological Economics 146, 125-135.

Scheffer, M., Straile, D., van Nes, E.H., and Hosper, H. 2001. Climatic warming causes regime shifts in lake food webs. Limnology & Oceanography 46, 1780–1783

Suding, K.N., Gross, K.L., and Houseman, G.R. 2004. Alternative sates and positive feedbacks in restoration ecology. TRENDS in Ecology and Evolution 19, 46-53.

Tomiyama, K. 1985. Two-stage optimal control problems and optimality conditions. Journal of Economic Dynamics and Control 9, 317-337.

Tomiyama, K., and Rossana, R.J. 1989. Two-stage optimal control problems with an explicit switch point dependence. Journal of Economic Dynamics and Control 13, 319-337.

Tuinstra, J., and van Wensem, J. 2014. Ecosystem services in sustainable groundwater management. Science of the Total Environment 485-486, 798-803.

Utz, R.M., Hilderbrand, R.H., and Boward, D.M. 2008. Identifying regional differences in threshold responses of aquatic invertebrates to land cover gradients. Ecological Indicators 9, 556-567.

Vörörsmarty, C.J., McIntyre, P.B., Gessner, M.O., Dudgeon, D., Prusevich, A., Green, P., Glidden, S., Bunn, S.E., Sullivan, C.A., Liermann, C.R. and Davies, P.M. 2010. Global threats to human water security and river biodiversity. Nature 467, 555-561.

Whiteman, M., Brooks, A., Skinner, A., and Hulme, P. 2010. Determining significant damage to groundwater-dependent terrestrial ecosystems in England and Wales for use in implementation of the Water Framework Directive. Ecological Engineering 36, 1118-1125.

Woodward, G., Perkins, D.M., and Brown, L.E. 2010. Climate change and freshwater ecosystems: impacts across multiple levels of organization. Phil. Trans. R. Soc. B. 365, 2093-2106.

WWF. 2014. Living the Planet. Species and spaces, people and places. Report 2014

Appendix

A detailed resolution of the two-stage optimal control problem

The social planner problem consists in the maximization of the social welfare, that is defined as the irrigators' private profits from groundwater pumping and the economic contribution of an aquatic ecosystem. The ecosystem depends on the groundwater resource and its functioning presents a shift to other status, once a critical threshold (H_C) is exceeded. The maximization (eq. 6) is a two-stage optimal control that needs to be solved through the resolution of two sub-problems with their associated Pontriagyny problems:

The first sub-problem can be stated as follows (eq. 7):

$$(A1)\ \ Max\ \ SP_2 = \int_{t1}^{\infty} e^{-rt} \cdot \left[\frac{1}{2k} \cdot W_2^2 - \frac{g}{k} \cdot W_2 - (C_0 + C_1 \cdot H_2) \cdot W_2 \right.$$
$$\left. + \xi \cdot (A \cdot (S_L - H_2)^2 + B \cdot (S_L - H_2) + C)\right] dt$$

$$s.t. \qquad \dot{H}_2 = \frac{[R + (\alpha - 1)W_2]}{AS}$$

$$H_2(t) \geq 0$$

$$H_2(t_1) = H_{\_1} \ \ and\ given$$

$$t_1 \quad free$$

The Hamiltonian of the social planner problem is expressed as:

$$(A2) \quad \mathcal{H}_2 = -e^{-rt}\left[\tfrac{1}{2k} \cdot W_2^2 - \tfrac{g}{k} \cdot W_2 - (C_0 + C_1 \cdot H_2) \cdot W_2 + \right.$$
$$\left. \xi \cdot (A \cdot (S_L - H_2)^2 + B \cdot (S_L - H_2) + C)\right] + \lambda_2 \cdot \left[\frac{R + (\alpha - 1) \cdot W_2}{AS}\right]$$

The first order conditions (FOC) of the Hamiltonian are:

$$(A3) \qquad \frac{\partial \mathcal{H}}{\partial W} = -e^{-rt}\left[\frac{1}{k} \cdot W_2 - \frac{g}{k} - (C_0 + C_1 \cdot H_2)\right] + \lambda_2 \cdot \left(\frac{\alpha - 1}{AS}\right) = 0$$

$(A4)\qquad \dfrac{\partial \mathcal{H}}{\partial H} = e^{-rt}(C_1 \cdot W_2 + 2 \cdot \xi \cdot A \cdot (S_L - H_2) + \xi \cdot B) \; = \; -\dot{\lambda_2}$

$(A5)\qquad \dfrac{\partial \mathcal{H}}{\partial \lambda_2} = \dot{H}_2 = \dfrac{[R + (\alpha - 1) \cdot W_2]}{AS}$

$(A6)\qquad \lim\limits_{t\to\infty} \lambda_2(t) = 0$

From the FOC (eqs. *A3* to *A6*) we can obtain the optimal value of the co-estate variable λ_2,

$(A7)\qquad \lambda_2 = \dfrac{AS}{(\alpha - 1)}\; e^{-rt}\left[\dfrac{1}{k} \cdot W_2 - \dfrac{g}{k} - (C_0 + C_1 \cdot H_2)\right]$

Equation *A7* is differentiated with respect to time (t), and then we can replace $\dot{\lambda_2}$ by the expression from equation *A4*,

$(A8)\quad - e^{-rt}(C_1 \cdot W_2 + 2 \cdot \xi \cdot A \cdot (S_L - H_2) + \xi \cdot B)$

$$= \; e^{-rt}\frac{AS}{(\alpha - 1)}\left(-r\left(\frac{W_2}{k} - \frac{g}{k} - (C_0 + C_1 \cdot H_2)\right) + \frac{\dot{W}_2}{k} - C_1 \cdot \dot{H}_2\right)$$

Rearranging terms and substituting $\dot{H}_2$ for its value (eq. *A5*) yields the following expression:

$(A9)\qquad \dot{W}_2 = r \cdot W_2 + \left(\dfrac{(\alpha - 1)}{AS} 2. k \cdot \xi . A - rkC_1\right) H_2 +$

$$\left(-r \cdot g - r \cdot k \cdot C_0 + \frac{R}{AS} \cdot k \cdot C_1 - \frac{(\alpha - 1)}{AS} \cdot k \cdot (2 \cdot \xi . A \cdot S_L + \xi \cdot B)\right)$$

that can be simplified as

$(A10)\qquad \dot{W}_2 = r \cdot W_2 + n \cdot H_2 + N$

where $m = \frac{(\alpha-1)}{AS}$, $M = \frac{R}{AS}$, $n = 2 \cdot \xi \cdot A \cdot k \cdot m - C_1 \cdot k \cdot r$, and $N = \big(-r \cdot g - C_0 \cdot k \cdot r + k \cdot C_1 \cdot M - m \cdot k \cdot \xi \cdot (2 \cdot \mathrm{A} \cdot \mathrm{S_L} + \mathrm{B})\big)$

A system of two ordinary differential equations can be established with the previous expression, equation *A10*, and with the FOC of the problem, equation *A5* ($\dot{H}_2 = m \cdot W_2(t) + M$):

$$(A11) \qquad \begin{pmatrix} \dot{W}_2 \\ \dot{H}_2 \end{pmatrix} = \begin{pmatrix} r & n \\ m & 0 \end{pmatrix} \begin{pmatrix} W_2 \\ H_2 \end{pmatrix} + \begin{pmatrix} N \\ M \end{pmatrix}$$

The solution of this system of differential equations can be calculated as the sum of the solution of the homogeneous system plus the particular solution. The first homogeneous equation of the system ($\dot{W}_2 = r \cdot W_2 + n \cdot H_2$) can be derived with respect to time (t) and substituting with the second homogeneous equation ($\dot{H}_2 = m \cdot W_2(t)$), we obtain the following expression:

$$(A12) \qquad \ddot{W}_2 - r \cdot \dot{W}_2 - n \cdot m \cdot W_2 = 0$$

The solution to this homogeneous differential equation is

$$(A13) \qquad W_2(t) = \bar{A} \cdot e^{tx_1} + \bar{B} \cdot e^{tx_2}$$

where the parameters $\bar{A}$ and $\bar{B}$ are equation's constants, and x_1, x_2 are respectively the roots of the polynomial equation ($x^2 - r \cdot x - n \cdot m = 0$).

By integrating equation *A12* we obtain the optimal solution of the water table problem ($H_2(t)$) under the first sub-problem (SP_2):

$$(A14) \qquad H_2(t) = \frac{m \cdot \bar{A}}{x_1} \cdot e^{tx_1} + \frac{m \cdot \bar{B}}{x_2} \cdot e^{tx_2}$$

The optimal solutions of the optimal extractions and water table level under the social planner problem (*SP2*) are:

$$(A15) \qquad W_2(t) = \bar{A} \cdot e^{tx_1} + \bar{B} \cdot e^{tx_2} - \frac{M}{m}$$

$$(A16) \qquad H_2(t) = \frac{m \cdot \bar{A}}{x_1} e^{tx_1} + \frac{m \cdot \bar{B}}{x_2} e^{tx_2} + \frac{-N + r\frac{M}{m}}{n}$$

The coefficients $\bar{A}$ and $\bar{B}$ are determined by using the sub-problem initial conditions, $t = t_1$ and $H_2(t_1) = H_{\_1}$. The value of $\bar{B}$ can be expressed as:

$$(A17) \quad \bar{B} = \frac{x_2}{m} . e^{-t_1 . x_2} . \left(H_{\_1} - \frac{m \cdot \bar{A}}{x_1} . \; e^{t_1 x_1} \; - \frac{r . N}{n} + M \right)$$

To determine $\bar{A}$, we use the transversality condition (eq. *A6*). This condition can only be satisfied when $\bar{A} = 0$ (similar to the result by Gisser and Sánchez (1980)).[5] With the value of these parameters, the optimal solutions of the problem are:

$$(A18) \quad W_2(t) = \bar{B} \cdot e^{t x_2} - \frac{M}{m}$$

$$(A19) \quad H_2(t) = \frac{m \cdot \bar{B}}{x_2} e^{t x_2} + \frac{-N + r \frac{M}{m}}{n}$$

Having the optimal values for both the state and the control variables we proceed to evaluate the value of the functional defined in equation (*A1*):

$$(A20) \quad \int_{t1}^{\infty} e^{-rt} \; f\big(W_2(t), H_2(t)\big) dt = \lim_{T \to \infty} \int_{t1}^{T} e^{-rt} \; f\big(W_2(t), H_2(t)\big) dt$$

It is demonstrated that it is a convergent integral because $x_2 < 0$ and $r > 0$. Solving the integral and the limit we obtain the optimal value of the sub-problem, $SP_2^*(W_2^*(t_1), H_2^*(t_1), t_1)$.

The second step of the 'two-stage' maximization method involves the resolution of the second sub-problem (*SP1*) with knowing the optimal solution of the first sub-problem (*SP2*). The first sub-problem (*SP1*) becomes:

[5] Realice that $x_2 < 0$.

$$(A21) \quad Max \;\; SP_1(W_1(t_1), H_1(t_1), t_1)$$

$$= \int_0^{t1} e^{-rt} \cdot \left[\frac{1}{2k} \cdot W_1^2 - \frac{g}{k} \cdot W_1 - (C_0 + C_1 \cdot H_1) \cdot W_1 \right.$$

$$\left. + \xi \cdot \left(\frac{(\sigma_2 - \sigma_1)(S_L - H_1)^2}{H_c^2} + \sigma_1\right)\right] dt + SP_2^*(W_2^*(t_1), H_2^*(t_1), t_1)$$

$$s.t. \qquad \dot{H}_1 = \frac{[R + (\alpha - 1)W_1]}{AS}$$

$$H_1(t) \geq 0$$

$$H_1(0) = H_0 \quad t_0 \;\; \text{and} \;\; H_0 \;\; given$$

$$H_1(t_1) = H_{\_1} \quad free$$

The Hamiltonian of the sub-problem (*SP1*) is expressed as:

$$(A22) \quad \mathcal{H}_1 = -e^{-rt} \left[\frac{1}{2k} \cdot W_1^2 - \frac{g}{k} \cdot W_1 - (C_0 + C_1 \cdot H_1) \cdot W_1 + \right.$$

$$\left. \xi \cdot \left(\frac{(\sigma_2 - \sigma_1)(S_L - H_1)^2}{H_c^2} + \sigma_1\right)\right] + \lambda_1 \cdot \left[\frac{R + (\alpha - 1) \cdot W_1}{AS}\right]$$

with λ_1 being the co-estate variable of the *SP1*.

The first order conditions (FOC) of the Hamiltonian corresponding with *SP1* ($\mathcal{H}_1$) are:

$$(A23) \quad \frac{\partial \mathcal{H}_1}{\partial W_1} = -e^{-rt} \left[\frac{1}{k} \cdot W_1 - \frac{g}{k} - (C_0 + C_1 \cdot H_1)\right] + \lambda_1 \cdot \left(\frac{\alpha - 1}{AS}\right) = 0$$

$$(A24) \quad \frac{\partial \mathcal{H}_1}{\partial H_1} = e^{-rt} \left(C_1 \cdot W_1 + 2 \cdot \xi \cdot \left(\frac{(\sigma_2 - \sigma_1)(S_L - H_1)}{H_c^2}\right)\right) = -\dot{\lambda}_1$$

$$(A25) \quad \frac{\partial \mathcal{H}_1}{\partial \lambda_1} = \dot{H}_1 = \frac{[R + (\alpha - 1) \cdot W_1]}{AS}$$

$$(A26) \quad H_1^* \; (0) = H_0 \;\; and \;\; H_1^* \, (t_1^*) = H_{\_1}$$

$$(A27) \quad \mathcal{H}_1[\; H_1^* \, (t_1^*), \;\; W_1^* \, (t_1^*), \;\; \lambda_1^* \, (t_1^*), t_1] = \frac{\partial \, \mathrm{SP}_2^*\left(W_{\,2}^{\,*}(t_1), H_{\,2}^{\,*}(t_1), t_1\right)}{\partial t_1}$$

From the FOC (eq. *A23*) we obtained the value of the co-estate variable:

$$(A28)\quad \lambda_1 = \frac{AS}{(\alpha-1)}\ e^{-rt}\left[\frac{1}{k}\cdot W_1 - \frac{g}{k} - (C_0 + C_1\cdot H_1)\right]$$

Equation $A28$ is differentiated with respect to time (t), and then we can replace $\dot{\lambda}_1$ by the expression from equation $A24$. Rearranging terms and substituting $\dot{H}_1$ for its value, yields the simplified equations:

$$(A29)\quad \dot{W}_1 = r\cdot W_1 + nn\cdot H_1 + NN$$

and

$$(A30)\quad \dot{H}_1 = m\cdot W_1(t) + M$$

with $m = \frac{(\alpha-1)}{AS}$, $M = \frac{R}{AS}$, $nn = -C_1\cdot k\cdot r + 2\cdot\xi\cdot k\cdot m\cdot\frac{(\sigma_2-\sigma_1)}{H_C^2}$, and $NN = -r\cdot g - C_0\cdot k\cdot r + k\cdot C_1\cdot M - 2\cdot\xi\cdot k\cdot m\cdot\left(\frac{(\sigma_2-\sigma_1)}{H_C^2}\cdot S_L\right)$.

A system of two ordinary differential equations can be established:

$$(A31)\quad \begin{pmatrix}\dot{W}_1\\ \dot{H}_1\end{pmatrix} = \begin{pmatrix} r & nn\\ m & 0\end{pmatrix}\begin{pmatrix}W\\ H\end{pmatrix} + \begin{pmatrix}NN\\ M\end{pmatrix}$$

The solution of this system of two homogeneous differential equations, which is solved just like previous sub-problem *SP2,* are the optimal extractions $(W_1(t))$ and water table level $(H_1(t))$ under the social planner problem 1, *SP1*:

$$(A32)\quad W_1^*(t) = \overline{CA}\cdot e^{ty_1} + \overline{CB}\cdot e^{ty_2} - \frac{M}{m}$$

$$(A33)\quad H_1^*(t) = \frac{m\cdot\overline{CA}}{y_1}\cdot e^{ty_1} + \frac{m\cdot\overline{CB}}{y_2}\cdot e^{ty_2} + \frac{(r\frac{M}{m} - NN)}{nn}$$

with $\overline{CA}$ and $\overline{CB}$, that are calculated with using the problem restrictions, are constants as: $\overline{CA} = \frac{H_C - H_0 e^{t_1 y_2} + \left(\frac{r \cdot \frac{M}{m} - NN}{nn}\right) e^{t_1 y_2}}{\frac{m}{y_1} \cdot (e^{t_1 y_1} - e^{t_1 y_2})}$ and $\overline{CB} = \frac{y_2}{m} \cdot \left(H_0 - \left(\frac{r \cdot \frac{M}{m} - NN}{nn}\right) - \frac{H_C - H_0 e^{t_1 y_2} + \left(\frac{r \cdot \frac{M}{m} - NN}{nn}\right) e^{t_1 y_2}}{(e^{t_1 y_1} - e^{t_1 y_2})} \right)$.